AN INTRODUCTION TO

PHYSICS

FOR TECHNICAL STUDENTS

HALER and STUART

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BY

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PREFACE

Physics is generally acknowledged to be essential to the equipment of every technical student, and the subject forms part of the curriculum of most Junior Day Technical Schools and Trade Schools.

In the following pages a scheme of the subject is developed, which forms a two-year course when two or three hours a week are devoted to the subject.

Since most technical students take Applied Mechanics and Electricity as separate subjects, the portions of Physics usually treated under these heads have been purposely omitted from this course and thus the danger of overlapping has been avoided.

Apart from these omissions the whole of the elements of the subject have been passed under review, and if the treatment of certain portions is brief, it is hoped that the breadth of outlook which the student derives from such a running survey will be considered sufficient compensation.

The experimental work has been confined to such as requires only the very simplest apparatus. Home-made appliances have a very distinct advantage over the more professional type at this stage of the student's work.

Sections I., II., and III. form a suitable course for the first year, and Sections IV. and V. may be completed during the second year.

As one of the authors is in the employment of the London County Council it is necessary in accordance with regulations to state that the London County Council is in no way responsible for the contents of this book.

P. J. H.
August, 1921. A. H. S.

A FOREWORD TO THE STUDENT

Many students of all ages and of all types show a marked tendency to regard certain subjects as being "useful," and others as being the reverse. No policy could have a more sinister influence on progress or place such a final limit on the student's actual usefulness in his vocation.

If any of the great discoveries or inventions which have revolutionised civilisation be examined and tracked back to its source, we invariably find a man working in some apparently quite useless field of research, with no object in view other than the acquisition of knowledge. *Utility* never enters his head, for the stage of the work engaging his immediate attention is such that no living man is in a position to say what is "useful" and what is not. The useless of to-day may be of paramount importance to-morrow.

When Volta in 1800 made his voltaic pile and obtained a feeble current of electricity, and Oersted in 1820 discovered the action of a current on a magnet; when Davy in 1821 demonstrated the power of a current to magnetise steel, and Faraday in 1831 showed that a current in one circuit could induce a current in another circuit, no one could foretell that these discoveries would lead to the production of a dynamo which, when rotated at Chelsea, could propel an electric train at Hampstead.

The contemporaries of the pioneers just mentioned may have said to them: "This is all very interesting, but what is the use of it?" If they had answered:—"In less than a century these principles will enable a man in London to speak to a man in Paris," they would have been laughed at.

Again, we may note that great discoveries and inventions have seldom if ever been made by one man. They are all

the results of the cumulative efforts of many workers. We generally associate the steam turbine with the name of Parsons, yet the turbine was only made possible by the development of the thermo-dynamics of steam carried out by Kelvin, Rankin, and others.

The course of Physics which is expanded in the following pages is intended for technical students, chiefly those associated with engineering. Yet its object is not to teach engineering; it is not primarily to teach *physics*. The object which the authors have had constantly in mind is to give to the student *the outlook of a physicist* upon engineering.

Education in any sphere does not consist in committing a number of facts to memory : it aims at producing an outlook on life or an attitude of mind. The mental attitude of a physicist is one of quantitative observation. He looks out on Nature and *measures* what he sees, and as physics deals chiefly with the sources of energy, it has, in the past, contributed much of value to engineering, and doubtless has more to offer in the future.

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AN INTRODUCTION TO PHYSICS

SECTION I.—THE PROPERTIES OF MATTER

CHAPTER I

UNITS OF MEASUREMENT

Units.—Physics is the science in which the properties of matter and the properties of energy are investigated quantitatively. All measurement involves a comparison between the thing to be measured and some standard quantity of the same nature, this standard being called the "unit."

It is very desirable that the number of different units employed in our measurements should be as small as possible, and it will be a further advantage if new units, as they become necessary, are made to depend upon those already in use.

Fundamental Units.—A fundamental unit is one which is necessarily independent of the others. For example, *length* is a measurement of a fundamental nature. In this country the unit of length defined by Act of Parliament is the *yard*. Multiples or submultiples of this are, of course, employed according to the magnitude of the thing to be measured. The *foot* (i.e., one third of a yard) is frequently employed in scientific measurement.

Another unit of length (defined by the law of France) is the *metre*. One hundredth part of this, called a centimetre, is the unit of length adopted in many physical measurements.

Mass, the amount of matter in a body, is another fundamental unit. The units of mass commonly used are the *pound* (in the British system) and the *gramme* (in the metric system).

Time is another fundamental idea of which we require a unit. In physics the *second*, which is $\frac{1}{86400}$ of a mean solar day, is universally used.

Derived Units.—The units just described deal with three quite independent ideas. If, however, we turn to ideas of *area* and *volume* it is unnecessary to define new independent units, for these quantities can be measured by the repeated determination of length. Thus for area we naturally use the *square foot* or the *square centimetre* as a unit, while the *cubic foot* or the *cubic centimetre* is a suitable unit of volume. These are called derived units.

There are a great many derived units, for experience has shown that almost anything may be measured in units derived from the fundamental units of length, mass, and time.

These derived units may be divided into two classes.

(1) Those using the foot, the pound, and the second, belong to the "F.P.S. system."

(2) Those using the centimetre, the gramme, and the second, belong to the C.G.S. "system."

Weight and Mass.—If we take a 1 lb. weight we may say that it contains one pound of matter. If we hang it on a spring balance the latter records the fact that this piece of matter "weighs" one pound.

Suppose we take the arrangement on to a lift. As the lift commences to rise the recorded weight of the body would be more than 1 lb., while as the lift was about to stop again the balance would show a weight of less than 1 lb.

Again, if we carried the balance and weight about the surface of the earth, we should have recorded more than 1 lb. near the poles of the earth, and less than 1 lb. near the equator.

Or yet another case. If the test were made on a high mountain or at the bottom of a deep pit, the balance would indicate a weight than less 1 lb.

It appears therefore that the *weight* of a body may vary from place to place, since it depends upon the attraction of the earth, which is known to vary in intensity. But it is contrary to sense for the amount of matter (or mass) to vary.

In using 1 lb. or 1 gramme as the unit of mass, the unit is defined as the weight of a standard *at a certain place and at a certain level*, such as sea level at Greenwich.

Engineers sometimes find it convenient to use the pound as the unit of weight and g lbs. as the unit of mass, g (the acceleration caused by the earth's gravity) being defined below.

Velocity.—Velocity may be defined as the rate of change of position. The units commonly employed are one foot per second, and one centimetre per second. A velocity of 1 foot per second indicates that the body is increasing its distance from some point of reference at the rate of one foot in every second.

Acceleration.—Acceleration is the rate of change of velocity. Thus, if a body at a particular instant of time is moving with a velocity of 5 feet per second, and after a period of 4 seconds its velocity has increased to 17 feet per second, the *amount* of change of velocity is 12 feet per second, and this change has taken place in 4 seconds. The change has, therefore, been at the *average* rate of 3 feet per second each second. This is usually written :—3 feet per second per second.

The units of acceleration commonly employed in physics are the foot per second per second, and the centimetre per second per second.

If a body be allowed to fall freely to the earth, its velocity constantly increases during its fall, owing to the persistent attraction which the earth exerts. Experiment shows that, falling freely from rest, a body acquires a velocity of about 32·2 ft. per second in the first second, and at the end of two seconds its velocity has become 64·4 ft. per second, and so on. Hence we see that the acceleration is at the rate of 32·2 ft. per second per second.

This value is generally denoted by the letter g. Measured in C.G.S. units its value is 981 centimetres per second per second.

Force.—Force is that which tends to overcome inertia. If one pushes or pulls at an obstacle with a force equal to the weight of one pound, a force is exerted, but the resistance to motion may or may not be overcome. The unit of force

is sometimes a force equal to the weight of one pound (or one gramme).

In many cases, however, it is convenient to define the unit of force as that which, acting on unit mass for unit time, gives it unit velocity. From this definition it is easy to see that $F=Ma$, where F is the force which, acting on M units of mass, produces in it a units of acceleration.

In the F.P.S. system this unit is called the *poundal*, and in the C.G.S. system it is called the *dyne*. It follows that there are g poundals in a force equal to one pound, and g dynes in a force equal to one gramme. For :—

F (in poundals) $=M$ (in lbs.) $\times a$ (in ft. per sec. per sec.), also F (in dynes) $=M$ (in grammes) $\times a$ (in cms. per sec. per sec.).

Work.—Work is done when resistance is overcome. The unit of work is done when unit force acts through unit distance. Thus the *foot-pound* is the work done by a force of one pound acting through one foot, and the *foot-poundal* is the work done by a force of one poundal acting through one foot.

An *erg* is the C.G.S. unit of work, and is the work done by a force of one dyne acting through a distance of one centimetre.

Energy.—Energy is that which is capable of doing work. Thus a rotating flywheel possesses *kinetic energy*. A weight raised above the earth's surface possesses *potential energy* in virtue of its position, and this energy is converted into kinetic energy when the body is allowed to fall to earth.

Heat is a form of energy which is converted into kinetic energy by a steam or internal combustion engine, and electricity is another form of energy which may be converted into heat by a resistance or into kinetic energy by an electric motor. Coal and petrol both possess *chemical energy*, which is converted into heat by combustion.

Energy may be measured in the same units as work, since the energy of a body is expressed by the amount of work it can do.

Collected Results.

	Units of the C.G.S. System.	Units of the F.P.S. System.
Length . .	Centimetre.	Foot.
Mass . .	Gramme.	Pound.
Time . .	Second.	Second.
Area . .	Sq. cm.	Sq. ft.
Volume . .	Cub. cm.	Cub. ft.
Velocity . .	Cm. per sec.	Ft. per sec.
Acceleration .	Cm. per sec. per sec.	Ft. per sec. per sec.
Force . .	Dyne.	Poundal.
Work . .	Erg.	Ft.-poundal.
Energy . .	Erg.	Ft.-poundal.

N.B.—The poundal and the foot-poundal are the true F.P.S. units of force and work respectively, although the pound and the foot-pound are often used by engineers.

Equivalent Values.

Length . .	1 ft.	$=30 \cdot 48$ cms
Mass . .	1 lb.	$=453 \cdot 6$ gms.
Area . .	1 sq. ft.	$=929$ sq. cms.
Volume . .	1 cu. ft.	$=2 \cdot 83 \times 10^4$ c.c.
Velocity . .	1 ft. per sec.	$=30 \cdot 48$ cms. per sec.
Acceleration .	1 ft. per sec. per sec.	$=30 \cdot 48$ cms. per sec. per sec.
Force . .	1 pdl.	$=1 \cdot 38 \times 10^4$ dynes.
Work and energy	1 ft.-pdl.	$=4 \cdot 21 \times 10^5$ ergs.

1 lb. $=g$ pdls. $=4 \cdot 45 \times 10^5$ dynes.

1 ft. - lb. $=g$ ft.-pdls. $=1 \cdot 236 \times 10$ ergs.

$g=32 \cdot 2.$

Ex. 1. Express a velocity of 50 miles per hour in feet per second and cms. per second.

$$\text{Velocity} = \frac{\text{Space}}{\text{Time}}$$
$$= \frac{50 \text{ miles}}{1 \text{ hour.}}$$
$$= \frac{50 \times 1760 \times 3 \text{ ft.}}{60 \times 60 \text{ secs.}}$$
$$= 73{\cdot}3 \text{ ft. per sec.}$$

Again $$\frac{73{\cdot}3 \text{ ft.}}{1 \text{ sec.}} = \frac{73{\cdot}3 \times 30{\cdot}48 \text{ cms.}}{1 \text{ sec.}}$$
$$= 2235 \text{ cms. per sec.}$$

Ex. 2. At a given instant a train is travelling with a velocity of 24 miles per hour. Five minutes later the velocity is 46 miles per hour. Express the acceleration in (a) miles per hour per hour, (b) feet per second per second, (c) cms. per second per second.

The train gains 22 miles per hour in one-twentieth of an hour. The average acceleration is therefore 440 miles per hour per hour.

The student should note that while it is possible for a train to accelerate at this *rate* for a few minutes, the influence of friction is such that the acceleration cannot be maintaned for anything like an hour and hence the train never attains a *velocity* of 440 miles per hour.

Now acceleration $$= \frac{440 \text{ miles}}{(1 \text{ hour})^2}$$
$$= \frac{440 \times 1760 \times 3 \text{ ft.}}{(60 \times 60 \text{ secs.})^2}$$
$$= \frac{440 \times 1760 \times 3}{3600 \times 3600}$$
$$= 0{\cdot}179 \text{ ft. per sec. per sec.}$$

Also $$\frac{0{\cdot}179 \text{ ft.}}{(1 \text{ sec.})^2} = \frac{0{\cdot}179 \times 30{\cdot}48 \text{ cms.}}{(1 \text{ sec.})^2}$$
$$= 5{\cdot}45 \text{ cms. per sec. per sec.}$$

Ex. 3. The average pressure of the atmosphere at sea level is 14·7 lbs. per sq. in. Express this in dynes per sq. cm.

$$\text{Pressure} = \frac{14{\cdot}7 \text{ lbs.}}{(1 \text{ in.})^2}$$

$$= \frac{14{\cdot}7 \times g \times 1{\cdot}38 \times 10^4 \text{ dynes}}{(2{\cdot}54 \text{ cms,})^2}$$

$$= \frac{14{\cdot}7 \times 32{\cdot}2 \times 1{\cdot}38 \times 10^4 \text{ dynes}}{6{\cdot}45 \text{ sq. cms.}}$$

$$= 1{\cdot}014 \times 10^6 \text{ dynes per sq. cm.}$$

Or approximately a million dynes per square centimetre.

Ex. 4. A weight of 7 lbs. is raised vertically through 4 ft. Express the work done in foot-pounds, foot-poundals, and ergs.

$$\begin{aligned} \text{Work done} &= 7 \text{ lbs.} \times 4 \text{ ft.} \\ &= 28 \text{ ft.-lbs.} \\ &= 28 \times 32{\cdot}2 \text{ ft.-pdls.} \\ &= 901 \text{ ft.-pdls.} \\ &= 901 \times 4{\cdot}21 \times 10^5 \text{ ergs.} \\ &= 3{\cdot}8 \times 10^8 \text{ ergs.} \end{aligned}$$

Exercises 1.

1. Convert the following measurements from inches to millimetres :—$1\frac{1}{32}$, $2\frac{1}{16}$, $3\frac{3}{32}$. Check the results graphically.

2. Convert the following measurements from millimetres to inches :—3·175, 4·762, 307·97. Check the results graphically.

3. Convert the following measurements from metres to feet :—1·1, 1·5, 3·9.

4. Convert the following areas from square centimetres to square inches :—1·2, 2·6, 9·7. Check the results graphically.

5. If 1 pound = 0·45359 kilogramme, convert the following readings in pounds to kilogrammes :—1·1, 2·5, and 4·7.

6. If 1 kilogramme = 2·2046 pounds, convert the following readings from kilogrammes to pounds :—5·1, 6·5, 7·7.

7. How many poundals are represented by the following pounds weight :—1·5, 2·8, and 3·7 ?

8. Express a velocity of 60 miles per hour in feet per second and centimetres per second.

9. A 'bus is travelling at the rate of 12 miles per hour. What is this velocity in feet per minute ?

10. Observations were taken to determine the velocity of a part of a mechanism and the following results were obtained :

Time in minutes . . .	0	1	2	3	4	5
Space passed over in inches .	0	3	6	9	12	15

Plot a space-time graph. If space divided by time = velocity (when the velocity is uniform), determine the velocity for each minute, and plot a velocity-time graph.

11. The previous experiment was repeated on another piece of mechanism and results were obtained as follows :—

Time intervals in minutes .	1	2	3	4
Space passed over in inches .	3·65	7·30	10·95	14·6

Plot a graph of time and space. What is the velocity in feet per minute for every minute ? Plot a graph of velocity and time.

12. Express a velocity of 30 miles per hour in feet per minute and centimetres per minute.

13. A wheel of 4 ft. 3 ins. diameter makes 200 revolutions per minute. What is the velocity of a point on the rim, in feet per second ?

14. At a given instant a train is travelling at the rate of 30 miles per hour and 5 minutes later the velocity is 35 miles per hour. Express the average acceleration (*a*) in miles per hour per hour, (*b*) feet per second per second, (*c*) centimetres per second per second.

15. A piece of steel is stressed to 2 tons per square inch. Express this stress in dynes per square centimetre.

16. The pressure in a boiler is 100 lbs. per square inch. Express this in dynes per square centimetre.

17. A weight of 4 pounds is raised through a vertical height of 8 feet. Express the work done in foot-pounds, foot-poundals and ergs.

18. It is noted that in 65 seconds a car has travelled one kilometre. Express the velocity in kilometres per hour and miles per hour.

CHAPTER II

PROPERTIES OF SOLIDS

Matter.—Matter is generally considered as that which possesses weight. It is conveniently divided into three classes, known as solids, liquids, and gases.

Solids.—A solid is that form of matter which may be submitted to compression without lateral support. Thus, iron being a solid, it is possible to take an iron cylinder, stand it on its base and place a weight on the top. The iron is now in compression, and, provided the weight is not excessive in relation to the dimensions of the cylinder, no appreciable deformation is produced.

Here we have a solid in compression without lateral support. Water, on the other hand, being a liquid, could not be treated thus. One cannot have a " cylinder of water " unless the water is supported by a vessel of some solid material. If we wish to compress a liquid or a gas it must be supported in a tube or other vessel.

Light and Heavy Solids.—Aluminium is spoken of as a "light" metal, and lead is said to be "heavy." Both these expressions are intended to give an indication of the weight of a piece of matter *in relation to its volume.* For a pound of aluminium weighs as much as a pound of lead, but the former occupies nearly $4\frac{1}{2}$ times the volume of the latter.

Specific Gravity.—The specific gravity of a body is its weight compared with the weight of an equal volume of something else. Thus it would be legitimate to say that the specific gravity of lead is $4\frac{1}{2}$, meaning that it is $4\frac{1}{2}$ times as heavy as an equal volume of aluminium. But if it were compared with iron, the specific gravity would be only about $1\frac{1}{2}$, whereas compared with platinum it would be about $\frac{1}{2}$.

For the numerical value of the specific gravity of a body to be of any practical use, it is necessary to have some standard substance with which all other bodies may be compared. Water has been selected for this purpose and hence we have :—

The Specific Gravity of a body is the weight of that body compared with the weight of an equal volume of water.

It will be seen in Chapter XV that this definition, to be complete, must state the temperature of the water (which should be at 4° C.), but the student may neglect this point at the present stage of his work.

Density.—*The Density of a substance is the weight of unit volume of that substance.*

It will be seen that whereas the specific gravity of a substance has a constant numerical value, the Density will vary with different units. Thus the specific gravity of copper is 8·93, which means that a piece of copper weighs 8·93 times as much as an equal volume of water.

The Density of copper is 555-lbs. per cub. ft. and 0·321-lb. per cub. in., and 8·93 grammes per c.c. We might add to these

VOLUME OF REGULAR SOLIDS.

Solid.	Dimensions.	Volume.
Cube . .	Length of edge $=l.$	l^3
Prism . .	Area of face $=A.$ Length $=l.$	$A\,l$
Pyramid .	Area of Base $=A.$ Vertical height $=h.$	$\frac{1}{3}\,A\,h$
Cylinder .	Diameter $=d.$ Length $=l.$	$\frac{\pi}{4}\,d^2\,l$
Cone . .	Diameter of base $=d.$ Vertical height $=h.$	$\frac{\pi}{12}\,d^2\,h$
Sphere . .	Diameter $=d.$	$\frac{\pi}{6}\,d^3$

by changing the units. The student should note carefully that specific gravities may be expressed by mere numbers without the mention of any units, but in expressing a density the units employed must be stated.

From the foregoing it will be clear that in the practical determination of specific gravities and densities it is only necessary to find the volume and weight of a specimen of the material. If the specimen is a regular solid such as a cube or a cylinder, the volume may be calculated from one or two simple measurements.

The table on p. 21 will be of use to those who are not yet familiar with the methods employed in finding the volumes of regular solids.

Determination of Weight.—The weight of a body may be determined in two ways :—

(1) By noting the deformation of a spiral spring when the latter is put into tension or compression by the body in question.

(2) By balancing it with standard weights.

In the first case the spring has usually been calibrated, and the weight is recorded by a needle passing over a graduated dial.

In the second case a pair of scales and a set of weights may be used, similar to those employed in many shops.

In either case the weight recorded is only an approximation and such methods should only be employed when a rough determination is required on a comparatively large quantity of matter, say a pound or two. For more accurate work a chemical balance should be used.

The Balance.—This instrument is similar in general principle to that of a pair of scales, but a high degree of precision is maintained in every detail of it. Gramme weights are generally employed in connection with a balance of this nature. The student should examine carefully a balance and a box of gramme weights and note their arrangement. In using a balance it is necessary to observe certain rules, as a balance is such a delicate piece of mechanism that it is easily damaged.

Rules to be Observed when Using a Balance.

1. See that the box of weights is complete.

2. See that the base of the balance is level as indicated by the spirit level or plumb bob. If not, adjust the levelling screws.

3. Never place anything on a pan or remove anything from it unless the beam is resting on its supports.

4. When operating the lever which raises the beam from its supports never make a "jerky" movement. Endeavour to bring the beam in contact with its supports when the needle indicates that the beam is horizontal.

5. Raise the beam and observe the swing of the needle. It should swing an equal number of divisions of the scale on either side of the centre mark. If it does not do this the balance needs adjusting by means of the screw provided for the purpose. Students should, however, remember that this adjustment should not frequently be necessary, and in any case it should only be made by a person of experience.

6. No substance which is at all likely to injure the balance pan should ever be placed on the bare pan. Such substance should be weighed on a watch glass of known weight.

7. It is convenient to place the body to be weighed on the left-hand pan of the balance. The weights can then be operated by means of the forceps in the right hand, while the left hand operates the lever which raises the beam.

8. Don't *guess* at suitable weights. Find one which is too heavy, after which every weight below this one should be placed, separately, on the pan and the beam raised. The weight is allowed to remain if the weight total is too small, and removed to the box if the weight total is too great.

9. Every weight should be either on the pan or in the box.

10. Never put weights on the pan carrying the body to be weighed. It is quite unnecessary.

11. The weights are correct when the needle swings an equal number of divisions on either side of the centre mark. Don't wait for the beam to come to rest.

12. Never leave the beam unsupported longer than is absolutely necessary.

13. Determine the total of the weights by examining the spaces in the box.

14. Replace the weights in the box, totalling them as you do so. The totals should of course agree.

15. Every balance is designed to carry not more than a certain load on each pan. This is often 250 or 500 grammes. Great care must be taken never to over-load a balance. Carelessness on this point may cause permanent injury to the balance.

Ex. 5. A solid cylinder of brass is 3·5 cms. long, and has a diameter of 1·37 cms. It weighs 43·873 grams. Find its density in grams per c.c. and in lbs. per cubic in. Also find its specific gravity.

$$\text{Area of circular base} = \cdot 7854\,(1\cdot 37)^2 \text{ sq. cms.}$$

$$\text{Volume of cylinder} = \cdot 7854\,(1\cdot 37)^2 \times 3\cdot 5 \text{ c.c.}$$

$$\text{Density} = \frac{\text{Weight}}{\text{Volume}} = \frac{43\cdot 873}{\cdot 7854\,(1\cdot 37)^2 \times 3\cdot 5} \text{ grams per c.c.}$$

$$= 8\cdot 5 \text{ grams per c.c.}$$

$$\text{Density} = \frac{8\cdot 5 \text{ grams}}{1 \text{ c.c.}}$$

$$= \frac{\dfrac{8\cdot 5}{453\cdot 6} \text{ lbs.}}{\dfrac{1}{16\cdot 39} \text{ cub. in.}}$$

$$= \frac{8\cdot 5 \times 16\cdot 39}{453\cdot 6} \text{ lbs. per cub. in.}$$

$$= \cdot 307 \text{ lbs. per cub. in.}$$

Since 1 c.c. of water weighs 1 gramme it follows that the specific gravity of any body is numerically equal to the density expressed in grammes per c.c.

Hence the specific gravity of the brass of which this cylinder is made is 8·5.

Irregular Solids.—The methods adopted for the determination of the density of irregular solids will be more conveniently considered in Chapter III.

Experiment 1.

Determine the density in grammes per c.c., and lbs. per cub. in. of the material composing any regular solids, such as cubes, cylinders, spheres, etc., which are available.

Exercises 2.

1. Determine the volume of the following cubes, dimensions being given in centimetres in each case :—1·9, 2·4, 4·3, 0·45.

2. Determine the volume of the following cylinders, each cylinder being 1″ long and the diameters as given :—0·1″, 1·2″, 3·3″, 4·5″.

3. Determine the volume of the following spheres, the diameters being given in inches :—0·1″, 0·4″, 2·5″, 8·7″.

4. An indiarubber stopper weighs 26·545 grammes and its volume is 18·195 cubic centimetres. Determine its density in grammes per cubic centimetre and its specific gravity.

5. A piece of steel 1″ wide, $\frac{1}{4}$″ thick and 12″ long weighs 0·85 lbs. Determine its density in lbs. per cubic inch, its volume in cubic centimetres, and its weight in grammes. What is the specific gravity ?

6. Determine the density in lbs. per cubic inch of the following pieces of mild steel.

Breadth in ins.	Thickness in ins.	Length in ins.	Weight in lbs.
1	$\frac{1}{4}$	12	0·85
$1\frac{1}{8}$	$\frac{5}{16}$	12	1·2
$1\frac{1}{4}$	$\frac{3}{8}$	12	1·59

7. Determine the density of the following pieces of mild steel, each piece being square in cross-section:—Length of edge, $\frac{3}{16}$″, $\frac{1}{4}$″, and $\frac{7}{16}$″. Each piece is 12″ long and weighs 0·12, 0·213, and 0·651 lbs. respectively.

8. Find the density of cylindrical bars 12″ long and of the following diameters and weights, $\frac{3}{4}''$ dia., 1·912 lbs. $\frac{13}{16}''$ dia., 2·245 lbs., and $\frac{7}{8}$ dia., 2·603 lbs.

9. The density of the following substances is given in lbs. per cubic foot. Determine the density in grammes per cubic centimetre :—Water 62·4, aluminium 161·7, antimony 417, and bismuth 613.

10. The mass of a cubic foot of water is nearly 1000 ozs. (998·6 ozs.), so it often said that a cubic foot of water weighs 1000 ozs. A litre of water weighs 1000 grammes. Determine the density of water in lbs. per cubic foot and in grammes per cubic centimetre.

11. The *relative density of a substance* is the ratio of the mass of the body composed of it to the mass of an equal volume of the standard substance. For the purposes of this question take the standard substance as water at 62° F., weighing 62·355 pounds per cubic foot. What is the relative density of the following substances, whose weights are given in lbs. per cubic foot ?—Aluminium, 160 ; brass, 530 ; copper, 555 ; mild steel, 480. What is another name for relative density ?

12. The average weight in lbs. of one cubic foot of various substances is given. Determine the specific gravity in each case. —Chalk, 156. Flint, 162. Pure cast gold, 1,204. Wrought iron, 480. Silver, 655.

13. A table is given showing the weight per square foot for material of the thicknesses shown. Determine the density in lbs. per cubic inch in every case.

Material.	Weight in lbs. per sq. foot.	Thickness in ins.
Steel . .	15·504	0·38
Iron . .	5·36	0·134
Copper . .	1·585	0·035
Brass . .	0·856	0·2

14. Determine the specific gravity of the following timbers, given their weight in lbs. per cubic foot :—

Alder, 42. Apple, 47. Ash, 45. Beech, 46. Birch, 41.

15. The specific gravity of the following liquids taken at 60° F. is given :—Nitric acid, 1·54. Sulphuric acid, 1·849. Pure alcohol, 0·794. Ammonia (27·90 per cent.), 0·891. Determine in each case the weight of a cubic foot.

CHAPTER III

PROPERTIES OF LIQUIDS

Fluids.—A fluid is the general class name given to any form of matter which *flows*. The class includes gases, vapours, and liquids.

Liquids.—A liquid is a form of fluid which has a definite volume. It takes the shape of the vessel into which it is put, but does not expand to fill the vessel as does a vapour or a gas.

Density of a Liquid.—In determining the density of a liquid the method already applied to solids is used. Two measurements have to be made:—1. The volume of the liquid in question must be obtained. 2. The weight of this volume of the liquid must be determined. The calculation is then made as in determining the density of a solid.

A liquid cannot of course be measured except in a containing vessel, and it is necessary that the measurements of volume and weight should both be made while the liquid is in the same containing vessel, since liquid transferred from one vessel to another always diminishes in quantity owing to some remaining on the "wet" sides of the former vessel.

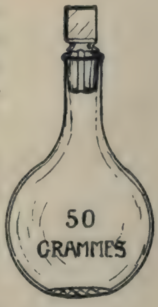

FIG. 1.

Specific Gravity Bottle.—Fig. 1 shows the common form of a specific gravity bottle. It consists of a thin glass flask, fitted with a ground stopper through which a fine hole has been drilled. If the stopper is removed and the bottle filled to the brim with a liquid, on gently lowering the stopper into position the

superfluous liquid flows through the fine hole in the stopper. The bottle is now exactly "full," and if it is made to hold a specified amount (usually 25 c.c.) we know the volume without further effort.

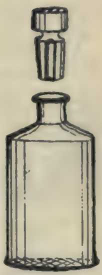

FIG. 2.

If a specific gravity bottle is not available a common 1 oz. stoppered bottle forms a good substitute, provided the stopper has been grooved down its side by means of a triangular file. (See Fig. 2.) In this case, however, the bottle must be calibrated for volume. An example will make this clear.

Ex. 6. A stoppered bottle weighs, when empty, 16·835 grms. Filled with pure cold water it weighs 43·728 grms. Find its capacity.

Weight of bottle and water . . .	=43·728 grms.
Weight of bottle alone . . .	=16·835 ,,
Weight of water alone . . .	26·893 ,,

Since 1 c.c. of water weighs 1 grm. the capacity of the bottle is 26·89 c.c.

Ex. 7. Using the bottle mentioned in Example 6, it was found that the weight when filled with lard oil was 41·457 grms. Find the specific gravity of the oil.

Weight of bottle and oil . . .	41·457 grms.
Weight of bottle alone . . .	16·835 ,,
Weight of oil	24·622 ,,

Now the result of Example 6 showed that the capacity of this bottle was 26·89 c.c.

Hence 1 c.c. of lard oil weighs $\frac{24 \cdot 622}{26 \cdot 89}$ grms. $= 0 \cdot 916$ grms. per c.c.

The specific gravity of lard oil is therefore 0·916.

Experiment 2.

Obtain a 1 oz. stoppered bottle and having cut a groove in the stopper calibrate it for volume by the method indicated in Example 6. If a specific gravity bottle of the pattern shown in Fig. 1 is to be used, it, too, should be calibrated in case the volume marked upon it is not quite correct.

Experiment 3.

Prepare a saturated solution of common salt in water. Find the density of this solution by the method indicated in Example 7.

Experiment 4.

Take some of the saturated solution of salt already prepared and mix with it an equal volume of water. Having thoroughly mixed these two liquids, find the density of the mixture. Is the density the mean of the density of water and that of the salt solution ?

Experiment 5.

Find the density of alcohol. (If alcohol is not available, common methylated spirit may be used. This is an impure form of alcohol.)

Experiment 6.

Mix alcohol and water together in equal parts and find the density of the mixture. Is the density the mean of those of alcohol and water ?

Experiment 7.

If the results of experiments 4, 5, and 6 appear at all confusing, test whether on mixing, say, 50 c.c. of water and salt solution, or water and alcohol, 100 c.c. of mixture is obtained. It will be a wise precaution to have a thermometer in the liquids to ensure that all measurements are made at the same temperature.

Level.—A " level " surface may be defined as one which is everywhere at right angles to the direction in which the earth's gravitational force acts. Where only small areas

are being considered a *level* surface may be regarded as being "flat" or "plane," but when we are dealing with extensive areas, such as the surface of a lake, it is necessary to remember that a level surface is bound to approximate to the shape of the earth and is therefore a portion of a spherical surface.

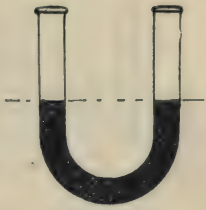

FIG. 3.

There is a popular saying that "water finds its own level." All liquids have this property, which amounts only to a tendency to flow in the containing vessel until the free surfaces are level.

Fig. 3 shows a U-tube containing a little liquid. The free surface is here broken into two owing to the shape of the containing vessel. Yet these two portions are each level when considered separately and they are both on the same level when considered together.

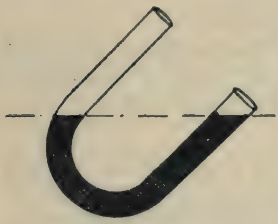

FIG. 4.

If the tube is tilted, as shown in Fig. 4, the level of the water is maintained although the vessel is now in a different position.

Liquids which wet the surface of a containing vessel show a tendency to creep up the side of the vessel. This results in the liquid being not quite level near the edge of the vessel.

Fig. 5 shows a few cases. It will be seen that if the vessel is large (as at A) the surface of the liquid is level except near the edge. As the surface becomes smaller, however, the effect is greater, until in very narrow tubes (as C) a liquid will appear to have a surface which is wholly curved.

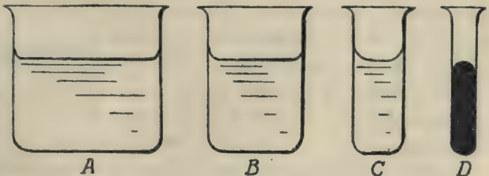

FIG. 5.

When making measurements of

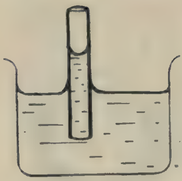

Fig. 6.

columns of liquids the measurement is always made from the *lowest* part of the curved surface in the case of liquids which wet the vessel.

Liquids which do not wet the vessel, such as mercury, have a curved surface which is convex (as D in Fig. 5).

In such cases measurements are made from the upper part of the curve.

Another modification of this phenomenon is observed when a tube (open at both ends) is placed in a vessel of liquid as shown in Fig. 6. The liquid creeps up the tube, the distance depending upon the diameter of the tube and the nature of the liquid.

Experiment 8.

By means of a few pieces of glass tube of different bores, observe the phenomena just described. Use a number of liquids, such as mercury, water, and various solutions.

Floating Bodies.—If a piece of wood is placed in a vessel of water it ultimately comes to rest with a portion of its volume immersed and the remainder above the water level. It is then said to be floating. Now since the immersed portion has, so to speak, made a hole in the water, the water which originally occupied this space has necessarily been obliged to find accommodation elsewhere. It has, of course, caused a raising of the level of the water.

Fig. 7 shows a sphere of wood floating in a vessel of water. The shaded portion of the wood indicates the amount which is immersed and the shaded portion of the water shows the amount of displaced water, the water having risen from A to B.

The *volumes* which these portions represent must of necessity be equal.

Now it is reasonable to suppose that if we push a quantity

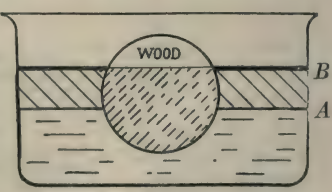

Fig. 7.

of water out of the position it naturally occupies, it will make an effort to return. Hence the *weight* of the displaced water forms a force which tends to push the wood out of the water. Since the wood is floating at rest it follows that :—

Weight of wood = weight of displaced water.

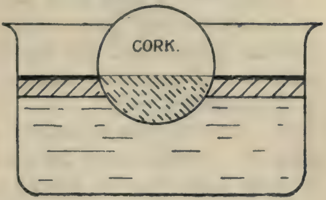

FIG. 8.

If a sphere of cork of the same size as that of the wood were placed in water, it would float with less of its volume immersed, for, since cork is specifically lighter than wood, a smaller volume of water is required to yield a force equal to the weight of the cork (see Fig. 8).

If a sphere of iron were placed in water it would pass into the water until the *whole* of its volume was immersed, and even then the weight of the displaced water would be less than the weight of the iron ; consequently the iron would not be supported but would sink to the bottom of the vessel.

Only bodies whose specific gravity is less than unity will float in water. Generally we may say, that bodies will float in a liquid provided the specific gravity of the body is less than that of the liquid.

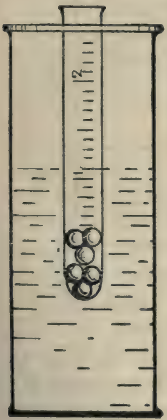

FIG. 9.

Experiment 9.

The Hydrometer.—Take a piece of glass tubing about 8 inches long and seal up one end. Place in it a few lead shot and above these a paper scale. (A roll of ordinary squared paper makes a suitable scale.)

If such a tube be placed in a vessel of water as shown in Fig. 9 the tube will float according to the law already given. That is, the immersed portion will displace a volume of water equal in weight to the weight of the tube, etc,

It is readily seen that the position of the level of the liquid as shown by the scale enclosed in the tube depends upon the specific gravity of the liquid. The denser the liquid the less will the tube need to sink in the liquid. Such a tube is called a hydrometer.

By varying the quantity of shot, a number of hydrometers may be constructed to suit liquids ranging from very low to very high specific gravities.

If the records of these instruments are compared with the specific gravity of the liquids obtained by means of a specific gravity bottle it is possible to replace the arbitrary scale of squared paper by a scale directly recording specific gravities.

Ex. 8. The following table shows the arbitrary readings of a glass tube hydrometer in liquids of known specific gravity. Prepare a scale of specific gravities for the hydrometer:

Specific gravity . .	1·	1·15	1·21.
Reading . . .	3·5″	5·1″	5·8″.

First plot a graph of these results.

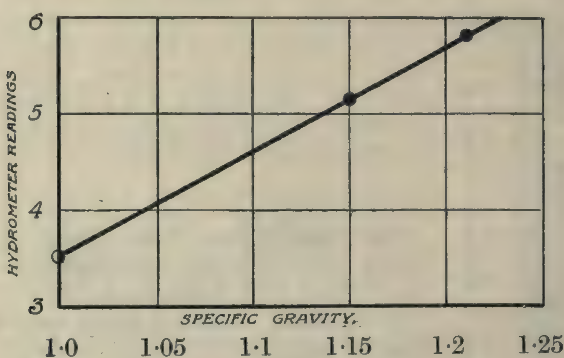

Fig. 10.

This is shown in Fig. 10, and it will be seen that in this case it is a straight line. From the graph it is possible to read the specific gravity corresponding to any scale readings. Thus we may compile a table as follows :—

Specific gravity.	Corresponding Scale Reading.
1·00	3·5″
1·05	4·05″
1·10	4·6″
1·15	5·1″
1·20	5·7″
1·25	6·25″.

If a scale is now prepared on which the above specific gravities replace the positions originally occupied by the corresponding scale readings we have a hydrometer which will record specific gravities directly.

It may be mentioned that a hydrometer may be made more sensitive by having a bulb blown at or near the bottom. Of course, the more *sensitive* we make the instrument, the smaller will be the range of its readings.

Determination of the Density of Irregular Solids.—We are now in a position to determine the density of solids of irregular form. Consider a piece of iron whose volume is, say, 10 c.c. If this be dropped into water it will displace 10 c.c. of water which will weigh 10 grammes. As we have seen, this displaced water will exert an upward thrust of 10 grammes on the iron. As the iron itself weighs more than this the force will be insufficient to float the iron. Nevertheless, if the iron were weighed while it was suspended in water it would be found to weigh 10 grms. less than its normal weight.

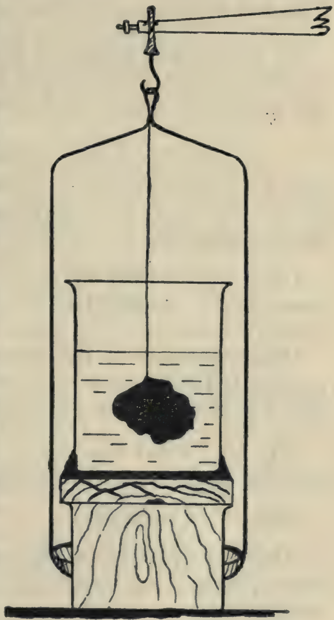

FIG. 11.

We can make use of this fact to obtain the volume of an

irregular body. Fig. 11 shows a simple arrangement for obtaining the weight of a body while suspended in water. A light wooden bridge spans the pan of an ordinary laboratory balance. On the bridge rests a vessel of water, and the body, suspended by a fine thread from the hook of the knife edge, is wholly immersed in the water. Its weight is now obtained in the usual way.

Ex. 9. A piece of lead weighs 257·928 grms. When suspended in water it weighs 235·295 grms. Find the density of lead.

Weight of lead	257·928 grms.
Weight of lead when suspended in water .	235·295 ,,
Upward thrust due to displaced water . .	22·633 ,,

As this is the *weight* of the displaced water and as 1 grm. of water has a volume of 1 c.c. it follows that the *volume* of the displaced water (and therefore of the lead) is 22·633 c.c.

$$\text{Now Density} = \frac{\text{Weight}}{\text{Volume}} = \frac{257{\cdot}928 \text{ grms.}}{22{\cdot}633 \text{ c.c.}}$$

$$= 11{\cdot}38 \text{ grms. per c.c.}$$

Experiment 10.

Find the density of a number of irregular solids, avoiding those which would dissolve in water.

Determination of the Density of a Powder.—Powders and granulated solids are best dealt with by means of the specific gravity bottle. An example will make the method clear.

Ex. 10. Find the density of sand, employing the specific gravity bottle used in Example 6.

Weight of sand taken . . . . 3·148 grms.

This was placed in the bottle, which was then filled with water, care being taken to remove air bubbles, and the whole weighed.

The weight was 45·523 grms,

Using the data of Example 6.

Weight of bottle and water . . .	43·728 grms.
Weight of sand	3·148 ,,
Adding we get	46·876 ,,
The actual combined weight was . .	45·523 ,,
Difference	1·353 ,,

This difference is due to the fact that the sand occupies space and therefore the bottle holds less water.

The weight of water displaced = 1·353 grms.

The volume of this water (and therefore of the sand) is 1·353 c.c.

$$\text{Density} = \frac{\text{Weight}}{\text{Volume.}}$$

$$= \frac{3{\cdot}148 \text{ grms.}}{1{\cdot}353 \text{ c.c.}}$$

$$= 2{\cdot}32 \text{ grms. per c.c.}$$

Determination of the Density of a Soluble Solid.—Solids which are soluble in water have applied to them one of the above methods, with the exception that a liquid in which the solid will not dissolve is substituted for water.

Ex. 11. A lump of salt weighs 18·412 grms. When suspended in alcohol of specific gravity 0·71 it weighs 14·108 grms. Find the density of the salt.

Weight of salt	18·412 grms.
Weight of salt suspended in alcohol . .	14·108 ,,
Upward thrust due to displaced alcohol .	4·304 ,,
Weight of displaced alcohol . . .	4·304 ,,

$$\text{Volume in c.c.} = \frac{\text{Weight in grammes.}}{\text{Specific gravity.}}$$

$$= \frac{4{\cdot}304}{0{\cdot}71}$$

$$= 6{\cdot}06 \text{ c.c.}$$

This being the volume of displaced alcohol is likewise the volume of the salt.

$$\text{Now density} = \frac{\text{Weight}}{\text{Volume.}}$$

$$= \frac{18{\cdot}412 \text{ grms.}}{6{\cdot}06 \text{ c.c.}}$$

$$= 3{\cdot}04 \text{ grms. per c.c.}$$

Hare's Apparatus.—Let two glass tubes be supported vertically with their lower ends respectively in two beakers containing different liquids as shown in Fig. 12, *A* and *B*.

Let the upper ends of the tubes be joined by a *T* piece (*C*) fitted with a tap. If the tap be opened to the air the liquid in the tubes will stand at the same height as that in its beaker (except for the creeping up effect called "capillary attraction," and this will be negligible unless the tubes are very narrow).

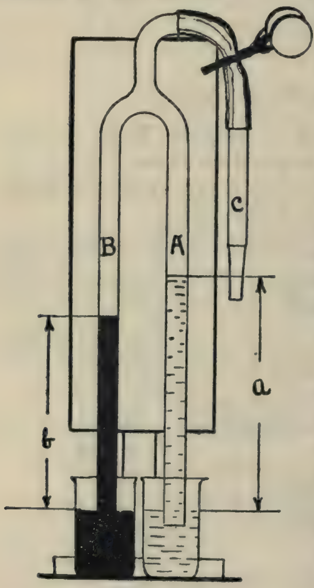

Fig. 12.

Suppose air is withdrawn from the tubes by suction through C and the tap closed. A certain amount of each liquid will be drawn into the tubes, and since the remaining air is free to pass from one tube to the other it follows that the two supported columns of liquid must balance.

If therefore the vertical height of each column above the level of the liquid in the beaker be measured it is possible to compare the density of the two liquids.

In Fig. 12 let the tube *A* dip into pure water and the tube *B* into a solution of salt. Since the latter solution is specifically heavier it follows that a shorter

column of it will be supported. It is easy to see that the specific gravity of the salt solution is given by $\frac{A}{B}$.

This is quite obvious if the two tubes are of exactly the same bore, but experiment shows that it is also true when the tubes are of different bore. It is not even *necessary* for the tubes to be of *uniform* bore. An explanation of this will be given in the next chapter.

Experiment 11.

Make a Hare's apparatus using two tubes of the same bore and find the specific gravity of salt water.

Repeat the experiment, using tubes of different bore, but the same salt water. Compare results.

In experiments of this nature the student should not be satisfied with a result obtained from a single reading. Several readings should be taken with columns of different heights and a mean value secured.

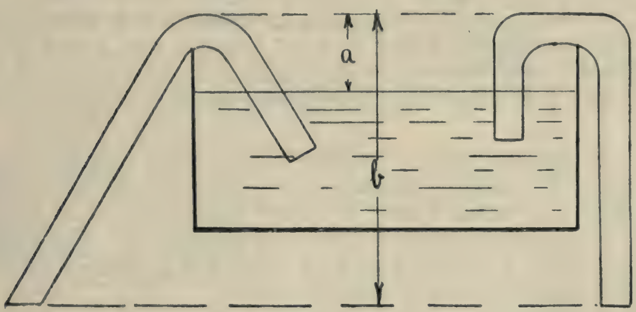

FIG. 13.

The Syphon.—Fig. 13 illustrates two syphons, which consist of bent pieces of tube each having one limb longer than the other. If such a tube be filled with a liquid and the short limb placed in a vessel of the liquid, the columns, being unequal in length, will not balance. The longer one, falling, will pull the shorter one after it, and thus liquid will

be drawn out of the vessel. It will be shown subsequently that it is the *vertical height* of such a column which is effective, and thus the two syphons illustrated would be of equal efficiency.

The difference in the lengths of a and b represents a column of liquid, the weight of which gives the effective suction.

Experiment 12.

Make a number of syphons and compare the amount of water which is taken from a vessel per minute with the value of $(b - a)$ in Fig. 12.

Exercises 3.

1. 25 cubic centimetres of copper sulphate solution were poured into a weighed evaporating basin. Determine the density of the solution from the following results :—weight of basin 55·152 grammes, weight of basin plus 25 cubic centimetres of copper sulphate solution, 83·805 grammes.

2. A steel ball 10 millimetres in diameter was placed in a burette containing water and its volume was found to be 0·35 cubic centimetres. Is this correct ? The weight of the ball was 4·47 grammes. Determine the true density of the ball.

3. A specific gravity bottle weighs 15·67 grammes and contains 66 cubic centimetres of water when full. Some vaseline was warmed until it flowed readily and then poured into the specific gravity bottle until it was full. The weight of the bottle was then found to be 72·372 grammes. Determine the density of the vaseline.

4. In determining the density of mercury by the "U" tube method the following readings were obtained, measurements being taken from the same level in each case. Height of water, 15·5 centimetres. Height of mercury, 1·14 centimetres. Determine the density of mercury.

5. An experiment was carried out to determine the density of mercury and the following results were obtained :—Weight of beaker 29·21 grammes. Volume of mercury weighed, 19

cubic centimetres. Weight of the beaker and the 19 c.c. of mercury 285·2 grammes. Determine the density of the mercury from these results and explain how you would carry out a similar experiment.

6. Sketch and describe Hare's apparatus and explain how you would determine the density of mercury with this apparatus. The following results were obtained with the apparatus :—Height of water 24·5 centimetres, height of mercury 1·8 centimetres. Calculate the density of mercury from these results.

7. A piece of iron weighed 26·734 grammes in air and 23·319 grammes when weighed in water. Determine its specific gravity.

8. The specific gravity of a piece of wood was required and the following experimental results were obtained :—

Weight of the body in air, 7·735 grammes.
Weight of the sinker in air, 26·735 grammes.
Weight of body plus the sinker in water, 16·05 grammes.
Weight of the sinker in water, 23·32 grammes.

Determine the specific gravity of the material and explain with the aid of sketches how you would carry out a similar experiment.

9. Hare's apparatus was used to compare the density of benzine with turpentine, and in carrying out the experiment the following readings were noted :—Height of benzine $11\frac{3}{8}''$, height of turpentine $9\frac{5}{16}''$. Compare the density of benzine with that of turpentine and the converse.

A further experiment was carried out with benzine and water and the following readings were taken :—Height of benzine 11·6″, height of water 10·2″. Determine the density of the benzine.

10. The following results were obtained after carrying out the experiment in question 1. Weight of the beaker holding copper sulphate 41·7 grammes. Weight of the beaker plus copper sulphate solution 155·13 grammes. Volume of the weighed copper sulphate solution 100 cubic centimetres. Determine the density.

At the completion of the test 100 cubic centimetres of water were added to the copper sulphate solution, and the weight of the beaker and solution, plus added water, was now 255·075 grammes. Determine the new density. Is the density the mean of the density of water and that of the copper sulphate solution ?

11. An experiment was carried out to determine the specific gravity of a wooden stopper and the following results were obtained :—

Weight of stopper and sinker in air . .	21·82 grammes.
Weight of stopper and sinker in water .	10·58 ,,
Weight of the sinker in air . . .	13·85 ,,
Weight of the sinker in water . . .	12·55 ,,

Determine the specific gravity of the stopper.

12. A test tube was taken and a small quantity of lead shot placed in it. A piece of squared paper was then placed in the test tube and the test tube was placed in various liquids and the height of flotation noted in each case. The results were as follows :—

Scale reading . .	0·313	0·9	1·5	1·55
Specific gravity . .	0·92	1·21	A	1·358

Plot a graph of specific gravities and scale readings and determine the specific gravity of " A."

13. An experiment was carried out to determine the influence of the bore of the tubes in Hare's apparatus and the results are as stated :—

With tubes of the same bore—

Height of water in the tube	5·5″
Height of alcohol	6·9″

With tubes of different bores—

Height of the water	5·75″
Height of the alcohol	7·37″

Determine the specific gravity in each case; does the bore of the tube affect the result in this experiment ?

14. An experiment was carried out to determine the effect of altering the bore of the tubes on the quantity of water delivered by various syphons. (See Fig. 13.)

First Experiment—

Bore of the tube, 2 millimetres.
Difference of vertical heights, 0·66″.
Outflow in cubic centimetres per minute = 75.

Second Experiment—

Bore of tube, 4·5 mm.
Difference of the vertical heights 1·5″.
Outflow in c.c. per min. = 142.

Third Experiment—

Bore of tube, 4·5 mm.
Difference of vertical heights, 1·75″.
Outflow in c.c. per min. = 412.

Carefully examine these results and state what deductions can be made.

15. An experiment was carried out to determine the specific gravity of glass and the weight of the glass in air was 2·85 grammes. The weight of the glass in water was 1·687 grammes. Determine the specific gravity of the glass.

CHAPTER IV

PROPERTIES OF GASES

Fluid Pressure.—If we consider a cylindrical vessel of diameter d, similar to that shewn in Fig. 14, containing a liquid to a depth of h, it is obvious that the volume of liquid is $\frac{\pi}{4}d^2h$, and if the density of the liquid is unity (as in the case of water, which weighs 1 grm. per cubic centimetre) this expression gives the weight of the liquid.

Now this weight is clearly supported by the base of the vessel, which has an area of $\frac{\pi}{4}d^2$; hence the pressure of the vessel is h units per unit area.

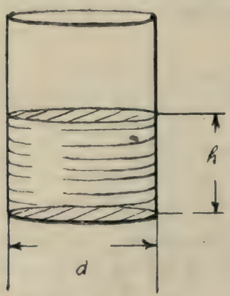

FIG. 14.

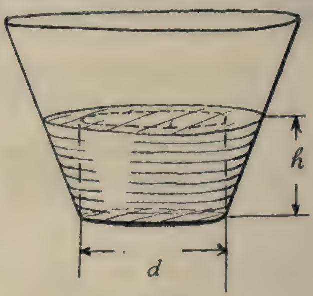

FIG. 15.

Now consider a vessel of the shape shewn in Fig. 15. It is clear that the base of the vessel supports a cylinder of water of the dimensions shown by the dotted lines, that is, the pressure on the bottom is the same as that in the case

just considered. The additional water is in this case supported by the sides.

Consider a point P which has a depth of x units of water above it. If a small hole could be pierced in the walls of the vessel at this point the water would issue normally to the wall of the vessel with a force proportional to the depth x.

We may say therefore that in this case at least the oblique sides of the vessel are subjected to a pressure which is directly proportional to the depth of water at the point considered and acts normally, that is, at right angles, to the surface of the vessel at the point.

Lastly, consider a vessel of the shape shown in Fig. 16. Here a concentric circle in the base supports a complete cylinder of water and the pressure on this is obviously the same as in the previous examples. The annular space round this circle has above it water varying in depth from o to h. Consider a given point. Here the pressure is again normal, that is, perpendicular to the surface. Furthermore, it is, as in the last example, proportional to the depth of water at this point.

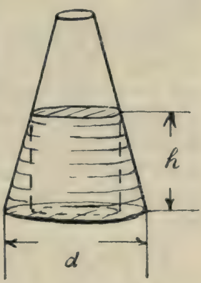

FIG. 16.

Now it is an established principle in mechanics that every force is met by a reaction equal in magnitude and opposite in direction. Hence at a given point the wall of the vessel must be pressing *downwards* with a force equal to the pressure of the water *upwards*. Hence if the downward pressure at this point is X units, this added to the column of water of y units immediately below it, makes a total pressure on the bottom of the vessel of $X+y$ units, which is equal to h.

It appears therefore that the pressure on the bottom of the vessel is everywhere the same, namely, h units per unit area. It obviously must be so, for if it were not, water would flow from the region of high pressure to one of lower pressure, until equality of pressure was established.

We have seen that the total pressure on the base of the vessel in Fig. 16 is the same as that represented in Fig. 14,

yet the *weight* of water in the conical vessel is less than that in the cylindrical vessel. This is sometimes a little confusing to a student, but it is similar in principle to the case of a common lever. Here a *small* force applied at one end becomes a *large* force at the other, but the work done at both ends is equal. The question of work has, so far, not entered into our consideration of fluid pressure.

Properties of Gases.—A gas is another form of fluid, but a gas can not only *flow*; it has also the special property of filling its containing vessel, irrespective of the quantity of gas or the size of the vessel; that is, there is no level surface terminating a quantity of gas such as we found in the case of a liquid.

Now if we put a pint of gas into a quart bottle the latter is only "full" in the sense that the gas is equally distributed all over the bottle. There is no space which contains more or less gas per unit volume than any other space. There is, however, nothing to prevent another pint of gas being put into the same bottle and a further pint after that. Thus a bicycle tyre is capable of holding a very variable quantity of air.

Experience shows, however, that the more air one pumps into a bicycle tyre the "harder" it gets: in other words, the higher the *pressure* of the air rises.

Another point which experience has established is that the air in a bicycle tyre will escape from a hole made in *any* part of the tyre. For a *liquid* to escape from a vessel it is necessary for the hole to be below the level of the liquid; but a gas exerts its pressure equally in all directions, hence it will pass through a hole in the top or side of a vessel as readily as through a hole in the bottom.

Atmospheric Pressure.—Gases, like other forms of matter, possess weight. It is true that their density is comparatively small, but it is by no means negligible. Air, for example, under normal atmospheric conditions, has a density of about 0·08 lbs. per cubic foot. In other words a cube of edge 2 ft. 4 ins. contains about 1 lb. of air.

Now just as a liquid exerts a pressure on the bottom of its containing vessel, so we may expect the air at the earth's surface to be affected by the weight of the air above it.

The case is not, however, quite so simple, for the air in the lower regions, being subject to the pressure due to the weight of the air above it, is compressed, and is therefore much denser than the air in higher regions. Liquids, on the other hand, are almost incompressible.

The Barometer.—In considering Fig. 14 we saw that the total pressure on the base of the vessel was equal to the weight of the water. As a matter of fact, to this amount must be added the pressure of the air on the surface of the water. It is clear, therefore, that the reaction of the force on the base of any vessel containing a liquid is acting vertically upwards and is equal to the pressure due to the liquid *plus* the pressure of the air.

Now if we could remove the atmospheric pressure from a portion of the liquid surface, the liquid immediately below the surface should be forced up. Such conditions can be partially obtained by placing a tube in a basin of water and sucking some of the air out of the tube. The water at once passes up the tube under the action of the force.

If we had a very long tube it is obvious that we might reach a limit where the pressure due to the column of water would be equal to the pressure of the air. No amount of suction could draw water up the tube beyond this limit.

This principle may be utilised to measure the pressure of the air. In order to avoid the use of very long tubes it is usual to employ a liquid of a density much higher than that of water: mercury, for example.

Experiment 13.

Obtain a piece of barometer tube about 33 ins. long. This tube has stout walls to give strength. The bore depends upon the quantity of mercury available, but for experimental purposes a bore of 3 or 4 millimetres is sufficient. One end of this tube should be sealed in a blowpipe flame.

This tube has now to be filled with mercury and certain precautions should be taken whenever mercury is used.

Precautions in the Use of Mercury.—Mercury is the only metal which is liquid at the ordinary temperature of the air. It readily amalgamates with certain other metals and should, therefore, be kept out of contact with other metals. In this connection it is a source of danger in a laboratory sink, as it is liable to amalgamate with (and destroy) the lead drainage pipes.

As mercury is very costly it is desirable to support any apparatus containing it in a deep wooden tray made for the purpose. If any mercury is spilled, it is then caught in the tray.

Mercury does not " wet " most surfaces, and consequently when dropped on to a flat surface, gathers itself up into numerous little globules which are not readily collected.

If in spite of precautions mercury *does* get into a sink, it may be removed by means of a small piece of zinc which has been dipped in dilute sulphuric acid. The mercury readily amalgamates with the zinc and is thus removed. If sufficient mercury is involved it may be obtained by dissolving the zinc in dilute sulphuric acid, after which the mercury may be washed with water and dried.

If mercury becomes mixed with moisture or dirt it may be cleansed by allowing it to pass through a funnel fitted with a filter paper in the apex of which a small hole has been pierced. The dirt adheres to the paper and the clean mercury passes through.

When a long tube has to be filled with mercury, the tube should if possible be supported in a slanting position, and in any case the mercury should be allowed to pass in very slowly and in small quantities at a time. Mercury has so high a density that any considerable quantity of it falling vertically down a long tube strikes the end with such force that there is a danger of the tube breaking.

Try to avoid air bubbles in a mercury column. With patience and a little gentle tapping they can generally be removed.

Returning to our barometer tube, when the tube is completely filled with mercury, the open end should be temporarily closed (this is generally done by the thumb of the right hand),

the tube inverted and the open end placed below the surface of a little mercury contained in a vessel, as shown in the left-hand portion of Fig. 17.

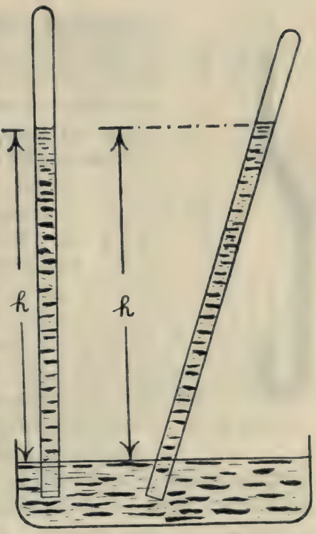

FIG. 17.

Now it is clear that no air can enter the barometer tube; hence there can be no air pressure on the surface of the mercury over which the tube is placed; but when all flowing has ceased the pressure must be equal at all points of the surface, so that there will remain in the barometer tube a column of mercury which exerts the same pressure as that which the air is exerting on the outside surface.

This column is generally in the neighbourhood of 760 mm. or slightly under 30 inches, and is shown by h in Fig. 17. It should be noted that the height of the mercury column is measured from the level of the mercury in the dish and not from the bottom of the tube. It is clear that the portion of the tube above the mercury column cannot contain anything except the vapour of mercury, which at ordinary temperatures is quite insignificant.

If the pressure of the air changes, mercury will flow into or out of the tube until the mercury column again balances the pressure.

Thus we speak of the pressure of the air being indicated by the "height of the barometer."

If a barometer tube be inclined as shown on the right hand side of Fig. 17 we find that the mercury remains at the same level. In other words, it is the *vertical* height of the column of mercury which records the atmospheric pressure.

Fig. 18 shows another type of mercury barometer, working on what is known as the "syphon" principle. Although

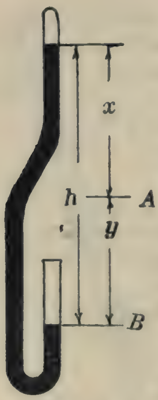

FIG. 18.

somewhat different in form the actual principle is the same. The air exerts a pressure on the open surface of the mercury at B and supports a column h units high. For purpose of graduation any point A is selected and scales proceed upwards and downwards from this point. The levels of the two surfaces of mercury are read on these scales and added together. Obviously $x+y=h$. When a permanent barometer is made for purposes where some degree of accuracy is required, it is necessary to *boil* the mercury in the tube to expel all air bubbles. This is an operation calling for skill and experience and should not be undertaken without suitable supervision.

A portable form of barometer is made in which the principle involved is quite different. A small flat vessel made of corrugated metal has the air extracted from its interior and is then sealed. The air pressure on the outside will depress its flat surfaces to a degree depending on the magnitude of the pressure. The movement is of course very small, but by a system of levers it is magnified into a suitable movement of a needle over a scale which may be graduated in any desired units. This instrument is called an "Aneroid Barometer."

Ex. 12. The normal height of the barometer is 760 mm. At sea level it seldom sinks below 725 or rises above 785 mm. Express these pressures in lbs. per sq. in.

Consider a tube whose sectional area is 1 sq. cm.

The volume of mercury in the three columns is respectively 76, 72·5, and 78·5 c.c.

Mercury has a density of 13·58 grms. per c.c. The weight of the column is therefore respectively :—

$$76 \times 13{\cdot}58 = 1{,}032 \text{ grms.}$$
$$72{\cdot}5 \times 13{\cdot}58 = 984 \quad ,,$$
$$78{\cdot}5 \times 13{\cdot}58 = 1{,}066 \quad ,,$$

These weights represent the pressure per sq. cm.

Now 1 kilogram per sq. cm. is equivalent to 14·22 lbs. per sq. inch. The pressures given above are, therefore, equivalent to :—

$$\frac{1032}{1000} \times 14 \cdot 22 = 14 \cdot 67 \text{ lbs. per sq. ins.}$$

$$\frac{984}{1000} \times 14 \cdot 22 = 14 \cdot 0 \quad ,, \quad ,,$$

$$\frac{1066}{1000} \times 14 \cdot 22 = 15 \cdot 17 \quad ,, \quad ,,$$

Ex. 13. What would be the normal height of a barometer in which water was used instead of mercury ?

Since mercury is 13·58 times as dense as water it follows that a column of water equal in pressure to that of mercury must be $760 \times 13 \cdot 58$ mm. or $33\frac{1}{2}$ ft.

It may be mentioned here that no suction pump can "lift" water to a height greater than this.

Relation between Volume and Pressure of a Gas.—We have already seen that when the pressure of a gas is increased (as in pumping up a bicycle tyre) its volume is diminished. It remains to find the relationship which exists between the pressure and volume.

Most operators of a bicycle pump will have noticed that when air is rapidly compressed it becomes hot, and since heat is likely to affect the volume of a gas, it is desirable that we should investigate changes one at a time. In the following experiment, therefore, steps must be taken to secure that the temperature remains constant.

Experiment 14.

Find the relation between the pressure and volume of a given mass of gas when the temperature remains constant.

Fig. 19 shows a suitable form of apparatus. It consists of a piece of stout glass tube bent in the form of a U, of which one limb is much longer than the other. The short limb is sealed.

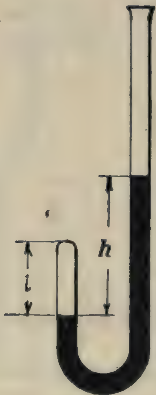

FIG. 19.

A little mercury is poured into the tube through the open end, and falling into the bend of the U imprisons a quantity of air in the short limb.

As more mercury is poured into the tube this air is compressed into a piece of tube of length l while the pressure exerted upon it is the atmospheric pressure *plus* a column of mercury of height h.

It is convenient to record pressure in terms of the length of a column of mercury instead of lbs. per sq. in. ; a transformation of units can readily be made when necessary.

Owing to the method of applying the pressure in this experiment the process is necessarily a slow one, and the very small quantity of heat which is produced by the small pressures involved has ample opportunity of passing to the surrounding air. It is not, therefore, necessary in this case to employ any special means of maintaining the temperature constant.

A number of readings should be made, commencing if possible with one in which the pressure is *below* the atmospheric pressure. This can be obtained by having the mercury level in the open limb *lower* than that in the closed limb. The height h would then be called a "negative head."

The following is a typical set of results :—

Reading of barometer, 765 mm.

Length l in mm. .	194	148	112	93	76
Length h in mm. .	−153	35	295	510	795

Now the length l may be regarded as proportional to the volume of the air. To the value of h the barometric reading must be added, and this will give the total pressure exerted on the volume of air, expressed in millimetres of mercury. Calling the volume v and the pressure p we have :—

v .	194	148	112	93	76
p .	612	800	1060	1275	1560

The student should use these results (or preferably those of his own experiment) to plot two graphs (i) to show the relation between p and v, and (ii) to show the relation between v and $\frac{1}{p}$.

He should also find the value of the product of p and v in each case. The best experimental work goes to show that the product of p and v is a constant. Another way of expressing this is to say that the volume varies inversely as the pressure. This law of Nature was discovered by Boyle in 1662, and its enunciation is known as Boyle's Law.

Boyle's Law.—If the temperature remain constant the volume of a given mass of gas varies inversely as the pressure.

Ex. 14. A quantity of gas is measured under a pressure of 834 mm. of mercury and found to have a volume of 15·7 cc. What volume would it occupy at a pressure of 760 mm. and at what pressure would its volume be 12 c.c.?

$$\text{Now } v = 15{\cdot}7 \text{ when } p = 834.$$
$$\therefore\ p\,v = 834 \times 15{\cdot}7 = 13{,}100.$$

But this product is a constant.

$$\text{Hence } V \times 760 = 13{,}100.$$
Where V is the required volume.

$$\therefore V = \frac{13100}{760} = 17{\cdot}2 \text{ c.c.}$$

$$\text{Also } P \times 12 = 13{,}100.$$

Where P is the pressure corresponding to a volume of 12 c.c.

$$\therefore P = \frac{13100}{12} = 1{,}092 \text{ mm.}$$

Pumps.—A pump is a mechanical device for transporting a fluid from one place to another. Pumps may conveniently be divided into two classes, rotary and reciprocating. In the former, *all* the moving parts rotate, while in the latter some at least of the moving parts "reciprocate"; that is, move to and fro along a straight line.

Valves are always associated with reciprocating pumps, a valve being a passage through which the fluid can flow only in one direction.

A common type of pump for dealing with air is the bicycle pump which is illustrated in Fig. 20. It consists of a cylinder with a plunger or piston. The latter consists of a disc of thin leather formed into the shape of a cup, the edges of which are turned towards the delivery end of the pump.

On the downward stroke the edges of this piston are pressed against the sides of the cylinder so that no air can pass. Thus the air is compressed into the lower part of the pump until the pressure is reached which is necessary to open the valve of the tyre. (This pressure of course depends upon the pressure of the air already in the tyre.)

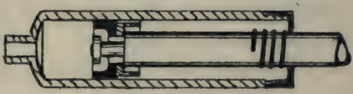

Fig. 20.

On the upward stroke, since no air can enter the pump from the delivery end (owing to the valve in the tyre) the pressure of the little air remaining in the pump soon falls, and is less than atmospheric pressure (that is, there is a partial vacuum). In consequence air pushes past the skirts of the piston, since these are not now being pushed against the cylinder walls, from the upper part of the pump which is open to the air. Thus one gets a cylinder full of air ready for the next compression stroke.

It is easy to see that as the pressure in the tyre gets greater, more and more of the compression stroke will have to be performed before the pressure rises sufficiently high to open the tyre valve, and the higher will be the pressure of the residual air left in the base of the pump.

A Common Air Pump.—A bicycle pump is used to pump

air *into* a receptacle, and may be called a "compressor." Fig. 21 shows a type of air pump which is used to extract air *from* a vessel and may be called a "*de*compressor."

Again it consists of a cylinder and piston. There is one valve at the end of the cylinder *A* and another in the piston *B*. These valves generally consist of a small hole over which a piece of oiled silk is stretched, this being placed on the side *away* from the vessel to be exhausted.

On the outward stroke, valve *A* opens and *B* closes. A partial vacuum is created in the cylinder and air flows from the vessel to the cylinder. On the return stroke *A* closes and *B* opens, and this air passes out into the atmosphere.

It is clear that as the pressure in the vessel falls, a higher and higher degree of vacuum has to be created in the cylinder before valve *A* will open, and thus the pump becomes less and less effective as the operation proceeds.

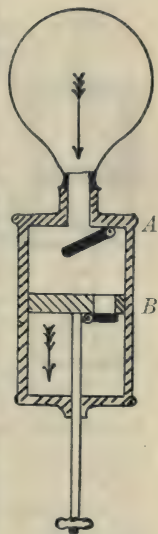

FIG. 21.

A Lift Pump for Water.—Fig. 22 shows a common form of water pump. There are two valves, both of which can only open upwards. They are often flap valves which operate after the manner of a door and normally remain closed by the operation of their own weight. One valve is placed at the bottom of the cylinder and the other is in the piston.

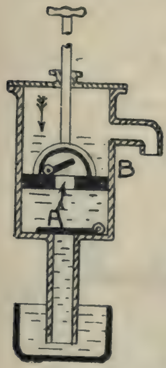

FIG. 22.

The diagram shows the piston on its down stroke. This causes the bottom valve to close and the water in the cylinder (being practically incompressible) opens the valve in the piston and flows through and ultimately passes out through the spout.

On the up stroke the valve in the piston is closed and as soon as a partial vacuum is created in the cylinder the lower valve opens

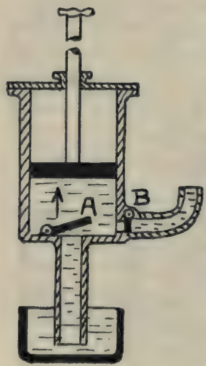

Fig. 23.

and water flows up (or is "lifted"). This flow is due entirely to the pressure of the air and hence it is impossible to "lift" water to a height greater than $33\frac{1}{2}$ feet (the height of a "water barometer").

Since such a pump could never produce a perfect vacuum the limit of lift in practice is very much less than this amount.

A Force Pump for Water.—When water has to be raised from a deep well or a mine, it is necessary to have a *force* pump. Such a pump is shown in Fig. 23, and it will be observed that there is no valve in the piston. The valves are fitted, one at *A* opening inwards and one at *B* opening outwards. The pump must be placed sufficiently near the water for the suction stroke to lift it.

The diagram shows the piston on the up stroke. *A* is open and *B* closed and water is flowing into the cylinder. When the piston reaches the top the cylinder is full of water.

On the downward stroke the compression causes valve *A* to close and valve B to open. Hence the water flows through *B* and up the outlet pipe, which may rise to any height whatever, provided the power supplied to the pump is sufficient to force the water out of the cylinder.

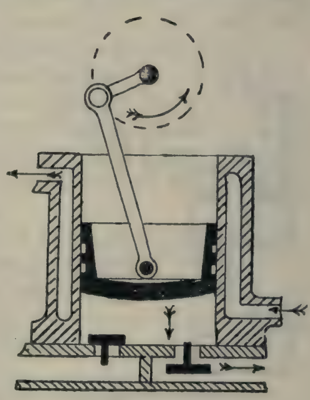

Fig. 24.

A Power-Driven Air Compressor.—Compressed air is used for so many purposes in modern engineering practice that it is necessary to have power-driven air compressors. A diagrammatic illustration of such a pump is shown in Fig. 24.

It is fitted with two valves of mushroom shape which lift

under air pressure against the action of a spring which normally keeps them closed. The piston is shown on the down stroke and air is being forced out of the cylinder into a receptacle. On the return stroke the outlet valve closes and the inlet valve opens, enabling air to be drawn into the cylinder ready for compression on the following stroke.

As these pumps often work at high pressures and speeds, the compressed air becomes very hot and is liable to give trouble. To meet this difficulty it is a common practice to place round the cylinder a jacket through which a stream of cold water is passed in the direction indicated by the arrows in the diagram.

Sometimes air is compressed in a pump of this nature, then cooled and passed into another pump which compresses it to a still higher pressure. This is called a "two stage compressor."

Diffusion.—Vessels made of unglazed earthenware and called " porous pots " are used in connection with the making up of electric cells. If one of these (preferably a small cylindrical one) is taken and fitted with a cork through which a glass tube passes it will be found that by blowing down the tube the cylinder appears to be "air tight." Indeed, it is possible to maintain a considerable pressure in such a vessel for some time.

If, however, such a vessel is filled with water it will be found that in an hour or two damp patches appear on the outer surface, showing that the walls of the vessel possess small pores through which water *can* pass, although the process is a very slow one.

If a porous pot, fitted with a cork and tube, be supported in an inverted position with the open end of the tube below the surface of a little water (preferably containing colouring matter) as shown in Fig. 25, the pressure of the gas within the porous pot cannot change without a visible movement in the water. That is, if the pressure rises, bubbles will pass from the tube through the water, while a fall of pressure will cause the water to pass up the tube.

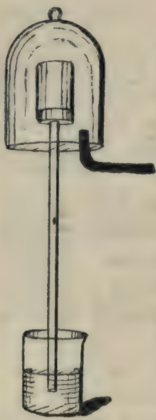

Fig. 25.

If a bell jar be placed over the pot as shown in the figure, we have air within and without the porous pot. Now pass hydrogen or other light gas (coal gas will answer) into the bell jar. Immediately bubbles begin to pass through the water, which seems to indicate that some of the hydrogen has found its way into the porous pot.

There is no question here of gas being forced through the walls of the vessel by pressure for the whole process is conducted at atmospheric pressure. The phenomenon is known as diffusion and depends upon a difference of *density*, and not a difference of pressure.

It is found that all gases can pass through the walls of a porous vessel at a rate inversely proportional to the square root of the density of the gas. Hence, in the case just given, hydrogen, having a density approximately one-fourteenth that of air, *entered* the porous pot at about $\sqrt{14}$ $(=3\frac{3}{4})$ the rate at which the air passed out. This accounts for the escape of some of the mixture in the form of bubbles.

After a time the porous pot will contain air mixed with a considerable quantity of hydrogen, and if the bell jar is removed this mixture will be less dense than the air outside, and consequently gas should pass *out* more quickly than air *enters*. This is indicated by the coloured water rising up the tube.

The *explanation* of the phenomenon of diffusion depends upon the molecular theory of gas and need not concern the student at the present stage of his work. The phenomenon itself is, however, too important to be neglected.

Experiment 15.

Fit up the apparatus shown in Fig. 25 and carry out the experiment just described. By replacing the gas jar by a jacket with its open end *upwards*, it is possible to try the effect of surrounding the porous pot with a heavy gas, such as carbon dioxide.

Exercises 4.

1. A cylindrical tank is 2′ 3″ in diameter and 2′ 6″ deep, and is full of water. Determine the pressure per square inch on the base.

2. A rectangular tank is 4′ 6″ long and 2′ wide, and $1\frac{1}{4}$″ deep. Determine the pressure per sq. in. on the base.

3. The absolute pressure in a condenser may be measured by means of a glass rod standing vertically in a bowl of mercury. When the upper end of the tube is connected with the condenser the mercury rises up the tube. If it rose 21″ on a day when the barometer stood at 758 mm., find the absolute pressure in the condenser.

4. The barometric pressure is 29·5″ of mercury. There is a vacuum in the condenser of 25″ of mercury. Find the atmospheric pressure and absolute pressure in the condenser in lbs. per sq. in.

5. Water is contained in a tank at the top of a building and the surface of the water is 30′ above ground level. Determine the height of a mercury column just to balance this head of water at ground level.

6. In a test on a small blower a "U" tube containing water was used to measure the difference of pressure between the air in and outside a large box. The following readings were taken :—0·395, 0·431, 0·464 inches of water. If the barometer was at 760 mm. determine the excess pressure inside the box.

7. The following readings of pressure and volume were obtained with a Boyle's Law apparatus. From these results determine the average value of the constant and plot a graph of pressures and volumes.

Pressure cms. of mercury.	132·2	122·6	114·5	105·8	101·2	89·8	73·7
Volume, in cc.	13·6	13·8	15·7	17	17·6	20	24·8

8. By means of the " U " tube determine the pressure of the coal gas supply from the main, in inches of water.

9. In a boiler test the force of the draught in inches of water was 0·38. What is the pressure in lbs. per sq. in.?

10. Give a sectional drawing of a bicycle pump. Assuming the pump described has 100% efficiency, measure the diameter and stroke of such a pump and state what volume of air at standard temperature and pressure it can supply each stroke.

11. Explain by the aid of sketches how any small glass syringe works.

12. Give a diagrammatic sketch of a self-filling fountain pen and explain its action.

13. In using a pipette explain why the liquid flows upward when the air is sucked away from the top of the tube. Why does the liquid stop running out of the pipette when the finger is placed over the top?

14. A cylindrical tin can is filled with water and the lid is soldered on. A small hole is then bored in one end near the outside rim. The can is placed in a horizontal position with the hole near the ground. Will all the water run out of the can? Carry out the experiment. Explain what must be done to get liquid out of a barrel after the tap has been fixed in position.

15. A " U " tube containing oil was used to measure the pressure inside a box as compared with that of the outside air; the air being taken from the outside air into the box and thence going to a gas engine. The oil in the " U " tube had a head of oil 0·16.

Determine the equivalent water column in inches, if the specific gravity of the oil was 0·9.

16. Take an ordinary leather "sucker." Calculate the maximum weight it can lift. Test this experimentally. Assuming that the leather is 2″ dia. determine the weight you would expect this to lift.

17. Take a tin can with a narrow neck such as a " Filtrate Oil " can, and clean it. Add a little water and boil the water

for some time. Remove the flame and insert a cork into the neck. What will happen to the can ? Try this experimentally.

18. Two surface plates as used in the metal workshop are placed with their finished faces together. What happens when you try and pull them apart ? Repeat this experiment with two pieces of plate glass and determine the force per sq. in. required to separate the two pieces.

19.

Barometer readings.	28	28·5	29	29·25	29·5	30	30·25
Corresponding pressure in lbs. sq. in.	13·75	14·00	14·24	14·37	14·49	14·73	14·86

The above table gives barometer readings and corresponding pressures in lbs. per sq. in. Plot the graph and determine a constant which multiplied by the barometer reading will give the corresponding pressure in lbs. per sq. in.

20. Plot the graph of height in feet above the earth's surface and pressure of the air in millibars. A " millibar " is a pressure of 1000 dynes per square centimetre.

Height in ft.	0	3280	6560	9840	13120
Mean pressure in millibars	1014	900	795	699	615

Convert the pressures in millibars into inches of mercury.

21. Certain boiling points for thermometer calibrations are given.

Pressures in ins. of mercury.	Temperatures in degrees F.
22·0	196·95
22·2	197·37
22·4	197·79
22·6	198·21

Convert the readings of pressures into millimetres of mercury, and those of temperature into degrees Centigrade. Express your results graphically.

22. Determine how many inches of mercury the following pressures in millibars are equal to :—

100 120 140 160 and 180.

23. A bicycle pump has a stroke of 11·5″ and a bore of 0·55″. What is the displacement of the pump ?

24. A bicycle tyre has a mean dia. of 26″. It is circular in cross-section and 1·14″ diameter. How much air at atmospheric pressure and temperature will it hold ?

25. 1 gramme per cubic centimetre = 62·43 pounds per cubic foot. Density of mercury at 0° C. = 13·5955 grammes per cubic centimetre.

Express the following barometer readings in millimetres as pressure in lbs. per sq. inch :—735 742 758 765 770 780.

26. In a table giving the thermal properties of water the following figures are given. Complete the table.

DENSITY.

lbs./(feet).^{3}	grammes/(centimetre).^{3}
62·37	
62·42	
62·43	

27. The following particulars are given of a single cylinder, single stage air compressor. Stroke in millimetres, 250 ; diameter of air cylinder, 240 mm. Determine how many cubic feet of air at atmospheric temperature and pressure should theoretically be delivered in one minute if the compressor makes 125 revs. per minute.

28. The following table shows the performance of suction pumps at a water temperature of 40° F. with the barometer at 29·92 ins.

Plot a curve of probable actual lift and vacuum in suction pipe in inches of mercury and state what lift you expect at 30″ vacuum.

Vacuum in suction pipe in ins. of mercury.	Theoretical lift in feet.	Probable actual lift in feet.
2	2·2	1·8
6	6·7	5·4
10	11·3	9·0
14	15·8	12·6
18	20·2	16·1
20	22·5	18·0
24	27·0	21·5
28	31·6	25·2
29	32·7	26·1
30		

29. Give sketches illustrating any type of pump used in motor-car engines to keep the cooling water for the engine jacket in circulation.

SECTION II.—PERIODIC MOTION

CHAPTER V

SIMPLE HARMONIC MOTION

Periodic Motion.—If a uniformly rotating steam engine is observed, it will be seen that the cross-head (which connects the piston-rod with the connecting-rod) moves to and fro within its guides with a velocity which is very far from uniform.

The movement is usually too fast for measurements to be made while the engine is running, but data for a space-time graph may be obtained by rotating the crank-shaft by hand through intervals of 30° (commencing with the crank at right angles to the axis of the cylinder), and recording the corresponding position of the cross-head. The following results were obtained on an engine having a stroke of 135 mm.

Crank Angle .	0°	30°	60°	90°	120°	150°
Distance of cross-head from initial position, in mm.	0	31·4	51·6	58·4	51·6	31·4

180°	210°	240°	270°	300°	330°	360°
0	−36·0	−65·3	−76·6	−65·3	−36·0	0

The outer dead centre is reached when the crank angle is 90° and the inner dead centre is reached with a crank angle of 270°. It will be observed that distances measured outwards from the initial position of the cross-head have been

regarded as positive and those are taken as negative which have been measured inwards from the initial position.

Fig. 26 shows a graph of these results, and deserves careful study. It will be observed that the slope of the graph (which indicates the velocity of the cross-head) is constantly changing. Also note that the distance moved from the initial position to the outer dead centre is considerably less than that from the initial position to the inner dead centre.

FIG. 26.

The graph possesses many features of irregularity. It will be seen that at the points marked A, B, and C on the graph the cross-head is in its initial position. At B the conditions are different from those of A in that the cross-head is travelling in the opposite direction.

At C, however, the cross-head is not only in its initial position, but all the conditions are exactly the same as those which obtained when the cross-head was at the point represented by A in the graph. In other words, if the readings were continued, the graph would repeat itself over and over again.

A graph which possesses this property is called a "periodic-graph" and the motion which it expresses is called "periodic motion."

Simple Harmonic Motion.—Fig. 27 shows the space-time graph of another body whose movement possessed fewer

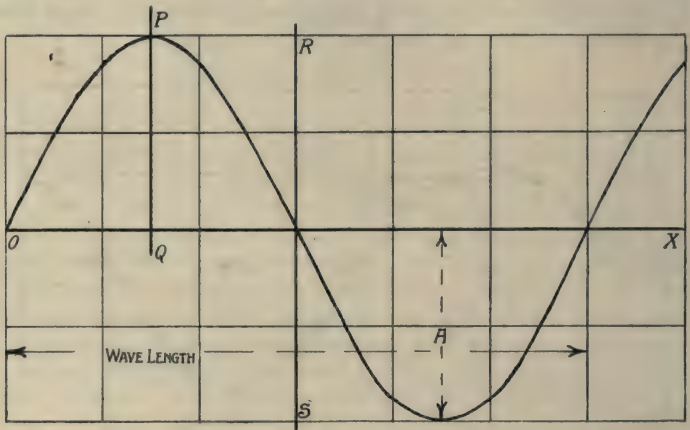

FIG. 27.

irregularities. The first half of the graph is symmetrical about the line $P\ Q$ and the whole graph is symmetrical about the line $R\ S$ except that the latter half is below the axis $O\ X$.

This graph is called a "sine curve" and any moving body whose space-time curve is a sine curve is said to possess "simple harmonic motion."

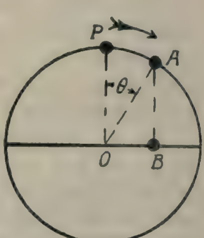

FIG. 28.

Ex. 15. Draw any circle having a diameter about 3″ long. Draw a horizontal diameter as shown in Fig. 28. Mark a point P vertically above the centre O and suppose that this point P travels round the circumference of the circle with

uniform velocity. Suppose another point travels to and fro along the horizontal diameter in such a way that it is always immediately above or below the point P. Thus when P has rotated through an angle of $\theta°$, that is, when it has passed to A, the point on the diameter has travelled from O to B.

Now make a table showing the corresponding values of $\theta°$ and the length of OB. Thus :—

Circle of 10 cms. diameter.

Value of θ in degrees.	Length OB in cms.
0	0
30	2·5
60	4·33
90	5
180	0
210	—2·5
240	—4·33
360	0

The point on the diameter moves to and fro with simple harmonic motion. Plot a space-time curve to prove this.

The Pendulum.—A thread is supported at A (Fig. 29) and carries a small heavy weight at B. Such an arrangement is called a "plumb line" and when at rest the thread will hang in a vertical direction. If we pull the weight to one side it is obliged to travel along the arc of a circle whose radius is equal to the length of the string. This necessitates the weight being raised above its initial level.

Thus in Fig. 29 the weight in travelling from B to C is raised through a vertical distance equal to B D. If the weight is now released the earth's gravitational force will tend to pull it down along the arc of the circle, and will impart

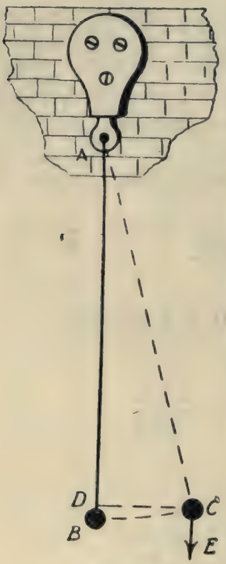

FIG. 29.

to it such a velocity that when B is reached the weight cannot stop, but is carried up an arc on the other side of the initial position until all its kinetic energy is expended. It now repeats the swing in the opposite direction.

We have here a "pendulum." If the thread possesses certain properties (among them *no weight*), and if the suspended weight has *no size*, the system is called a "simple pendulum;" otherwise it is called a "compound pendulum." For most practical purposes a small metal weight attached to a piece of thread or cotton may be regarded as a simple pendulum.

Experiment 16.

Suspend a 5 or 10 gramme weight by a length of about 40 or 50 cms. of cotton. The point of suspension is best secured by clamping the cotton between two strips of wood.

Now measure the length of the pendulum from the point of support to the centre of the weight. Call this length l.

The pendulum should now be made to swing *over a small arc* and the time required for it to make, say, 20 complete swings noted. (A *complete* swing is a swing to *and* fro.)

If this time is divided by the number of swings we obtain the "period" (expressed in seconds), called t.

Now change the weight, using a 20 or 50 gramme weight, but keeping the length as nearly as possible unchanged. Repeat the experiment and ascertain whether the magnitude of the weight has any influence on the period.

Experiment 17.

Set a pendulum swinging and record the time taken to accomplish 10 swings. The pendulum will now be swinging over a smaller arc, owing to friction. Without touching it,

record the time taken for a further 10 swings and repeat the record until the movements of the pendulum are too small to observe.

Is the period of a pendulum affected by the length of the arc over which it swings ?

Experiment 18.

Using lengths from 25 cms. to 100 cms. compile a table showing the corresponding values of the length l and the period t of a pendulum.

Plot a graph showing the relation between t and l.

Plot another graph showing the relation between t and $\sqrt{l}$.

It can be shown mathematically that :—

$$t=2\pi\sqrt{\frac{l}{g}}$$

where t and l have the meaning already given to them and g is the acceleration due to gravity.

g has a value of 32·2 ft. per sec. per sec. or 981 cms. per sec. per sec. l must be expressed in feet or centimetres, according to which unit is employed for the value of g.

Torsional Pendulum.—Another example of periodic motion similar to that of the pendulum just described is furnished by a heavy cylinder of metal suspended on a wire as shown in Fig. 30.

This is called a "torsional pendulum." If the metal cylinder is rotated through a small angle (about an axis passing along the wire) it will cause the wire to twist. This torsion of the wire will exert a force tending to rotate the cylinder in the opposite direction. When the cylinder reaches its initial position its acquired velocity causes it to twist the wire in the opposite direction and thus a rotational oscillation is set up.

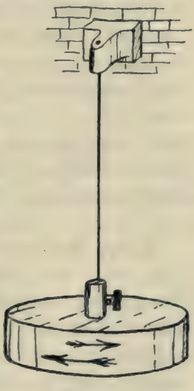

FIG. 30.

Experiment 19.

Construct a torsional pendulum and find its period t. Does the weight or size of the cylinder or the length or diameter of the suspending wire have any influence on the value of t? (Students who are interested in the mathematical or mechanical side of the subject will find a fuller treatment of this subject in the authors' *Second Course in Mathematics for Technical Students*, Chapter viii. It may be shown that

$$t=2\pi\sqrt{\frac{I}{c}}$$

where I is the moment of inertia of the cylinder and c is the value of the restoring couple due to the torsion of the wire when the cylinder is rotated through 1 radian.)

The Balance-Wheel.—Lastly, as an example of periodic motion, me ntion maybe made of the "balance-wheel." This consists of a delicately-balanced wheel, which in rotating has to overcome the resistance of a spring. This resistance brings the wheel to rest, and the action of the spring rotates the wheel in the opposite direction. It "over-shoots the mark" so to speak, as in the previous cases, and the spring pulls it back again. Thus an oscillatory motion is established.

The regularity of the periodic motions just considered has been applied to the purpose of controlling the movement of time-keepers. The common pendulum was first used for this purpose by Galileo during the latter part of the sixteenth century.

The torsional pendulum has been used in certain forms of clocks during recent years. The balance-wheel is employed in watches and portable clocks.

Wave Motion.—If a rope is secured to some elevated position and the free end pulled moderately tight and shaken, "waves" appear to run along the rope towards the fixed end. The form of the waves in the rope is shown in Fig. 31.

Now it is clear that the material of which the rope is made cannot be travelling along towards the fixed end, otherwise there would be an accumulation of material at that point.

A little thought should make it clear that any particle in the rope merely moves up and down on a path similar to that shown as XY in Fig. 31. Hence the *wave* travels along the length of the rope, but any particle of the rope merely moves to and fro along a comparatively short path at right angles to the direction in which the wave is moving.

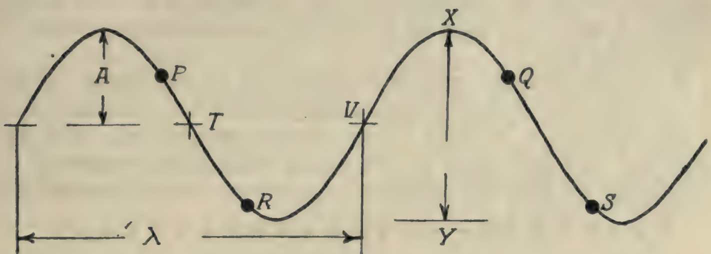

FIG. 31.

Transverse Wave Motion.—Such a wave propagation as that just described in a rope is called a transverse wave motion, because the particles of the medium conveying the wave are moving in a direction transverse (or at right angles) to the direction of the wave motion.

Another example of transverse wave motion is furnished by the waves formed on the surface of a liquid. The sea, viewed from the end of a pier, often has the appearance of flowing rapidly in a certain direction, but if a cork is dropped on to the surface of the water it usually rides up and down only, while the waves themselves pass on. Again, wind forming waves on the surface of a river often gives it the appearance of flowing up-stream. In this case the waves are travelling in one direction while the water itself is travelling in the opposite direction.

Longitudinal Wave Motion.—When a train consisting of a large number of trucks with link couplings is travelling with uniform velocity the links are all in tension. If the engine-driver now shuts off steam and applies the brakes to his engine

for a moment, the velocity of the engine is reduced and the first truck in the train bumps into the rear of the engine, and rebounds. This action impairs the velocity of that truck and the second truck bumps into the rear of the first and so the bumping process is carried right through the train.

Such a motion is another form of wave motion, but since the motion of the particles of the medium is in the same direction as the motion of the wave it is called longitudinal wave motion.

If a series of little balls were suspended on threads of equal length at equal distances apart, they would have the appearance of the row of dots in the upper part of Fig. 32. If now the end ball were set swinging after the manner of a pendulum so that the plane of its motion was the same as

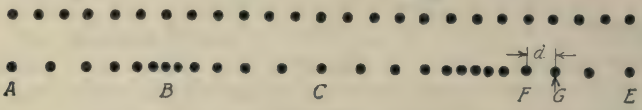

Fig. 32.

that containing the centres of the balls, it would bump into the second ball and set it swinging, and this would convey the motion to number three and so forth, very much after the manner of the bumping trucks in the train.

Now suppose at a particular moment the row of balls were photographed instantaneously, they would present an appearance similar to that in the lower part of Fig. 32. It will be seen that instead of the balls being equidistant they are crowded together at certain points, such as B, and spread out at others such as C.

A crowded group, such as B, would *appear* to travel throughout the row, just as the crest of a wave appears to travel over the surface of water, but actually each ball moves over a very short path only. For example, the ball at F has its position of rest at G and its next movement will be back to G, and then an equal distance beyond G.

Amplitude and Wave Length.—In both forms of wave motion the maximum distance which a particle moves from

its position of rest is called the "amplitude." Thus in Fig. 31 A is the amplitude and in Fig. 32 d is the amplitude.

Any two particles which are in similar positions and are about to perform the same set of motions are said to be in the same "phase." Thus in Fig. 31 P and Q are in the same phase, so also are R and S. In Fig. 32 A and C are in the same phase. It may be noted that in Fig. 31 the particles T and V are in the same relative positions, but they are not in the same phase because one is about to move upwards and the other downwards. For a similar reason A and B in Fig. 32 are not in the same phase.

In either case the distance between any point and the next one in the same phase is called a "wave-length," and is denoted by the Greek letter lambda (λ). Thus in Fig. 31 the wave-length is indicated by PQ and in Fig. 32 $\lambda = A\ C$.

Frequency and Velocity.—The number of complete waves which appear to pass a given point in unit time is called the frequency. It is usually denoted by N. If V is the velocity of the wave motion it is easy to see that $V = N\lambda$.

Exercises 5.

1. An engine has an infinitely long connecting-rod and the following data for a space-time graph were obtained (commencing with the crank at right angles to the axis of the cylinder).

Crank angle	0°	30°	60°	90°	120°	150°	180°
Distance of crosshead from the initial position—							
	0	0·5	·866	1	·866	·5	0
Crank angle	210°	240°	270°	300°	330°	360°	
Distance, etc.	−·5	−·866	−1	−·866	−·5	0	

Plot a space-time graph. Compare this graph with Fig. 27, which illustrates the graph of a simple harmonic motion.

2. An engine has a crank 1′ 0″ and a connecting rod 3′ 0″ long. Commencing measurements with the crank at right angles to the axis of the cylinder the following results were obtained :—

Crank angle	0°	30°	60°	90°	120°	150°	180°	210°
Distance of the crosshead from the initial position—	0	·45	·725	·825	·725	·45	0	—·55

Crank angle	240°	270°	300°	330°	360°
Position, etc.	—1	—1·175	—1	—·55	0

Plot a space-time graph. Compare the graph with Fig. 27. Describe any difference you note between this graph and the graph in the preceding question.

3. Draw a horizontal line OX on a piece of paper. Take a thin piece of wood and fix a pin at the point O and 2″ away from O fix a drawing-pin at P, carrying a piece of cotton, to which a weight is attached. Place the pin at O in the line OX and revolve the wood OP, noting every 30° where the plumb line fixed at P intersects the line OX. Call the distance OM. Tabulate the results as follows :—

Angle OP makes with OX	0°	30°	60°	90°	120°
Distance OM measured from O	2″	1·732	1″		

Complete the graph of the two results and compare the graph with Fig. 27.

4. A steel ball 10 millimetres in diameter was placed in a concave lens of 7·709 centimetres radius and given a gentle push. It was found to make 10 oscillations in 6·5 seconds. What was the periodic time?

The periodic time can be calculated from the following formula :—$T=2\pi\sqrt{0\cdot434R}$.

Where R=the radius of the concave surface minus the radius of the steel ball and all measurements are taken in feet.

Does the periodic time as calculated agree with the time as determined from the experiment? Carry out similar experiments.

5. A spiral spring had a weight Z placed on the end and was given a pull to set it in oscillation. The spring made 48 oscillations in 10 seconds. Determine the periodic time. After completing the experiment it was found that F lbs.

extended the spring $\frac{x}{12}$ feet. If $R = \frac{F \times 32 \cdot 2}{\frac{x}{12}}$ (poundals), determine the periodic time from the formula, $t = 2\pi\sqrt{\frac{Z}{R}}$

$Z = 5$ lbs., $F = 0 \cdot 5$ lbs., and $x = 0 \cdot 05$. Does the calculated periodic time agree with the experimental time ?

6. A "U" tube containing water to a depth of 6 inches was slightly displaced from its mean position. The water was set in oscillation and made 20 oscillations in 11 seconds. What is the periodic time ? Calculate the periodic time from the formula $T = \frac{\pi}{4}\sqrt{a}$ where a is taken in feet and represents the depth of the water.

7. Weights of 10, 20, 50 and 100 grammes were suspended by equal lengths of cotton (44 cms.). The following table gives the time of swing in each case. Determine the time of swing and state what conclusions could be drawn if a number of other experiments gave similar results.

Weight in grammes . .	10	20	50	100
No. of swings . . .	42	42	42	42
Time to complete swings in seconds	60	60	60	60

8. The swings per minute of a pendulum were recorded, and are as follows :—First minute 44 swings, second minute 44 swings, third minute 44 swings. It was noted that the arc of swing became smaller. Is the time of swing altered by the length of arc of swing ? Repeat this experiment.

9. A number of pendulums of different lengths were selected and the following results were obtained :—

Length of pendulum in centimetres :—	20	40	60	80	100
Period of oscillation in secs. :—	0·85	1·3	1·6	1·8	1·88

Plot a graph of these results and state what appears to be the general effect of shortening the length of the pendulum.

10. Taking the formula, $t=2\pi\sqrt{\frac{l}{g}}$, determine the time of oscillation or period of the pendulums in the preceding question. Plot a graph of t and l.

11. Place some water in a "U" tube and by blowing down one limb set the water in oscillation. Determine the periodic time of the oscillation. Carry out a number of experiments and see whether the following formula gives the same results. $t=\frac{\pi}{4}\sqrt{a}$ where a is the length in feet of the water column.

12. Take a straight steel spring or a uniform thin strip of wood; fasten one end in the vice. Set the rod in vibration and determine the periodic time. Fasten to the free end of the rod a small camel hair brush which is just arranged to touch a piece of cardboard. Dip the end of the brush in ink and move the cardboard rapidly and uniformly along in the direction of the length of the spring. Give a sketch of the curve produced. Fill a funnel having a narrow outlet with sand and suspend as a pendulum. What figure is traced out if a piece of cardboard is placed just below the pendulum? Is the figure of uniform thickness?

13. Repeat the above experiment but now move the cardboard at a uniform rate at right angles to the plane of the swing of the pendulum. What diagram is traced out in this case?

14. How many vibrations per minute would a pendulum 39″ long make at a place where $g=32{\cdot}2$ feet per sec. per sec.?

15. Determine the length in inches of a seconds pendulum, given the value of g in British units as $32{\cdot}1740$ feet per sec. per sec.

16. A clock loses 5 minutes per day; the pendulum should beat seconds. Determine the alteration in length required to make the clock keep correct time. Take g as equal to $32{\cdot}174$ feet per sec. per sec.

SECTION III.—SOUND

CHAPTER VI

VIBRATION AND THE MUSICAL SCALE

Propagation of Sound.—If one watches the firing of a distant gun, the flash of the explosion is seen and after an appreciable interval the sound is heard. Since it is known that the flash and the noise occurred together, it follows that light travels very much more quickly than sound, and that the latter, at any rate, does not travel instantaneously.

If between the gun and the observer there is an obstacle (a building for example) the flash of the gun is not seen, but the sound is still heard. From this it follows that light cannot ordinarily travel round an obstacle, whereas sound does do so.

Lastly, if the ear is placed to the earth it often happens that a distant sound is heard which would have been inaudible through the air. It is seen, therefore, that sound can travel through solid matter (like the earth) as well as, and perhaps better than it travels through the air.

Experiment has shown that matter of some kind is necessary for the propagation of sound, for if an electric bell is hung in the bell-jar of an air-pump the sound reaches the outside very readily so long as the bell-jar contains air, but as a vacuum is produced the intensity of the sound rapidly diminishes.

Vibration.—If we observe very closely a piano wire after it has been struck it will be seen that it is "vibrating;" that is, it is moving very rapidly from side to side. The note or sound which the wire is producing continues to be audible so long as the vibration continues, and it is obvious to the

most casual observer that the volume of the sound is more or less directly proportional to the extent of the movement of the wire in its vibration.

Now the beating of this wire against the air sets up a series of " waves " similar in many respects to the wave of bumping trucks in the train illustration of the last Chapter.

In other words longitudinal wave motion is set up in the air, and it is by this means that the vibration is conveyed to our ear and the sensation of sound produced. The wave-length is certainly long, often several feet, and the frequency is very high, usually several hundreds per second.

The Velocity of Sound.—We have already seen that although sound travels very rapidly its progress is not instantaneous. Before considering the measurement of its velocity it is advisable to determine whether all kinds of sound travel at the same rate. Does a " high " note, for example, travel more or less rapidly than a " low " note ?

If a band is playing at a distance it is obvious that we hear the notes some time after they are actually played. Now if different notes had different velocities, notes which were played together would not be heard together and the harmony would be upset. Such is not the case, and it is therefore safe to assume that sound of different frequencies has the same velocity.

To obtain a value for the velocity of sound, in air, it is only necessary to record the time occupied by sound to travel over a measured distance. Thus if we record with a stop-watch the interval between the flash of a gun at a known distance and the moment when the noise of the explosion becomes audible, we have the necessary data.

Since the sound is propagated by the actual vibration of the air particles it follows that if there is any wind it will affect the result. Naturally one chooses for such an experiment a day when there is as little wind as possible, but even so it is impossible to ensure perfectly still air.

This difficulty is readily overcome by having two guns, one at each end of a measured line. The velocity of sound is

then determined in one direction and immediately afterwards in the opposite direction.

Thus any assistance which was afforded to the sound waves by an air movement in one determination would be an equal hindrance in the other; hence the mean of the two results gives a reliable value.

It is found that the velocity of sound in air is affected to some extent by the presence of moisture, and also by the temperature, but it may be taken to be about 330 metres per second or 1,080 feet per second.

The velocity of sound in water may be determined in a similar manner by having two boats at a known distance. A bell below the surface of the water is struck by a lever motion which causes a flash of light at the same moment. The sound is received by observers in the other boat with the aid of a submerged listening tube.

It is found that sound travels through water with approximately four times its velocity in air.

In both the experiments just described it is assumed that no time is occupied by the passage of the light between the observers. Although this is not strictly true it can be shown that light has the enormous velocity of 186,600 miles per second, and hence the time occupied by the light in passing over a distance of a mile or two is quite beyond the recording limit of a stop watch.

Vibration of a Stretched String.—If a weight of a few pounds be suspended by a piece of string about a foot long, the string, being in tension, takes up the form of a straight line. If it is plucked slightly the weight immediately tends to restore the straight line, and in doing so gives the string a transverse velocity which carries it beyond the required position, and thus a periodic motion is established.

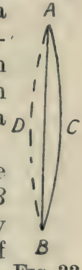

FIG. 33.

The string, which normally occupies the straight line AB in Fig. 33, alternately takes up the position ACB and ADB. The actual distance is, however, relatively much smaller than that shown in the diagram. If the weight is increased (other things remaining un-

changed) the string will be given a greater velocity, that is, its "frequency" will be raised.

The Monochord.—The monochord, shown in Fig. 34, is a suitable instrument for investigating the laws governing the vibration of a stretched string or wire. It consists of a strip of wood (or a box open at one end) about a yard long and a few inches wide. This is fastened to a bench, and a wire or string, secured at A, passes over a fixed "bridge" B, then over a movable bridge C, and finally over a pulley D to carry a scale-pan on which a weight W may be placed. Various pieces of string and wire should be tried, but avoid very stiff wire which it is difficult to bend. The bridges should be made of hard wood, or if soft wood is used a piece of steel wire should be fixed along the ridge.

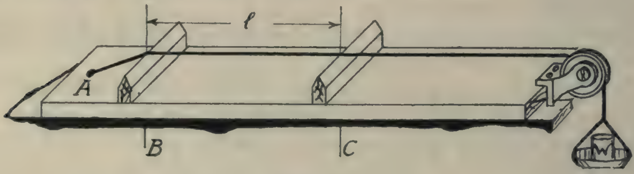

Fig. 34.

Experiment 20.

Fit up a monochord, and keeping the load W constant, find the effect of altering the length l between the bridges. Next keep l constant and vary the load W. Lastly, keep load and length constant, find the effect of substituting a thinner or a thicker wire of the same material.

Musical Note.—The ear readily detects the difference between a noise and a musical note. Physically the difference is that a musical note is propagated by a longitudinal vibration of regular frequency and wave-length. A noise is also propagated by a longitudinal vibration, but both the frequency and the wave-length are irregular.

A musical note possesses three qualities: (i) pitch (i. e., the note is "high" or "low"), (ii) intensity (i. e., the note is "loud" or "soft"), (iii) timbre or quality.

The pitch of a note depends upon the frequency of the vibration causing it. The more rapid the vibration the "higher" the note. The intensity of the note depends upon the amplitude of the vibration. The conditions deciding timbre will be considered later.

The Musical Scale.—In the upper part of Fig. 35 is shown a portion of the keyboard of a piano. This section of notes is called an "octave" because it contains eight white notes. The letters which are used to designate these eight notes are shown, as is also the notation used in music.

If two notes have the same frequency they are said to be in "unison"; if they have different frequencies the ratio of the frequencies is called the "interval" between the notes.

At the bottom of Fig. 35 a row of numbers is given which show the *relative* frequencies of the eight notes of the octave shown. It will be seen that the interval between C and D is 27/24ths or 9/8ths. Between D and E it is 10/9ths and between E and F 16/15ths. Following on we have intervals of 9/8ths, 10/9ths, 9/8ths, and 16/15ths.

Fig. 35.

Certain intervals are given names. For example, an interval of 9/8ths is called a "major tone," while an interval of 10/9ths is called a "minor tone." The interval of 16/15ths is called a "diatonic semitone."

The numbers given express only *relative* frequencies, and these always hold good. The *actual* frequency of a note varies slightly. The frequencies of all notes are fixed when

the actual frequency of one is given, and for this purpose the C nearest the middle of a piano manual is generally used. It is called "Middle C."

In physics the frequency of middle C is generally taken as 256 vibrations per second. Modern concert pitch makes it 276. The French standard pitch has 261, while the Stuttgart pitch (adopted by the Society of Arts) makes it 264. "Kneller Hall," a pitch largely adopted by military bands, gives middle C a frequency of 269 vibrations per second.

Knowing the pitch of one note and the interval between this and another note it is easy to find the pitch of the latter. Thus the frequency of G (concert pitch) is $\frac{36}{24} \times 276 = 414$ vibrations per second.

The scale just described is called the "Natural Scale." A slightly modified scale is now used in music. This is called the "Tempered Scale." It makes the interval of a "half tone" the same in all parts of the scale. Thus the following intervals are equal: C and $C\sharp$, E and F, $B\flat$ and B. The following frequencies are useful for comparison.

	C	*D*	*E*	*F*	*G*	*A*	*B*	*C′*
Natural Scale	256	288	320	341·3	384	426·6	480	512
Tempered Scale	256	287·4	322·5	341·7	383·6	430·5	483·3	512

Experiment 21.

It can be shown that if a wire is in tension the frequency (N) of the note which it emits when it vibrates freely is given by :—

$$N = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

where l = length of wire in cms.
T = tension of wire in dynes.
m = mass of wire per unit length (*i.e.* grammes per c.m.).

Using the monochord the student should test the frequency of a few notes on a piano or any other musical instrument which is available.

A few trials may be necessary to get the most suitable wire and tension.

Demonstrate that the "interval" of an octave is 2. In other words, the frequency of any note is double that of the note one octave below it.

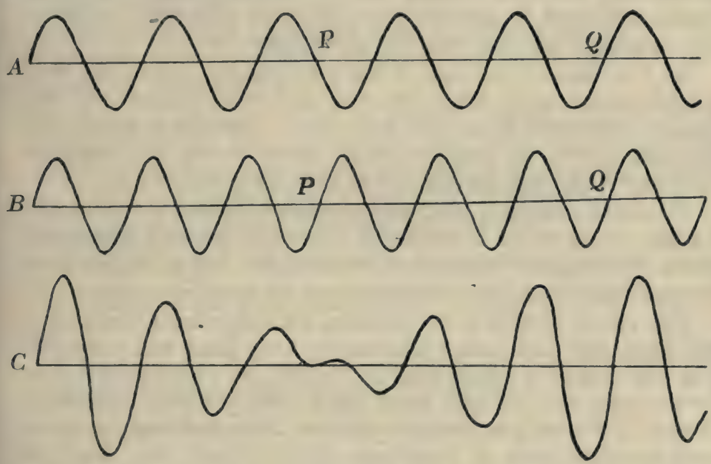

Fig. 36.

Beats.—If any two different notes are sounded together it is obvious that the particles of air which convey the sound to the ear are performing two different vibrations simultaneously. The actual movement of any particle of air is the resultant of two different movements.

When two waves of nearly the same wave length are combined a phenomenon known as "interference" occurs.

Consider Fig. 36. Since it is difficult to represent a longitudinal wave graphically, the diagram shows transverse waves only, but the student will readily appreciate that what is true of one is true of the other, in this respect.

The graphs A and B show waves of nearly the same wave length, A being slightly the longer. It will be observed that at the beginning they are "in phase," but owing to the difference of wave length the two waves gradually get out of phase until at the point P they are in opposite phase. That is to say, a particle conveying such a pair of waves would at this point be called upon to move equally in opposite directions, the result being no movement at all. At Q the waves are back in phase again, the longer having completed five waves while the shorter has completed six.

At C is shown the sum of the two graphs A and B. Frequent points have been selected and the distance of each graph measured from the zero line. Distances above this line are taken as positive while those below are negative. The algebraic sum provides the data for graph C.

It will be observed that the amplitude (which is responsible for the intensity or loudness of a note) is greatly augmented when the original vibrations are in phase, but it sinks almost to nothing when the vibrations are in opposite phase.

The effect of this is to produce an unpleasant beating on the ears, the note being alternately very loud and very soft. Fig. 36 makes it clear that a "beat" or loud effect occurs every time the shorter wave gains one complete vibration. Hence if we are producing a note of 256 vibrations a second and another note is produced which causes one beat per second it follows that the frequency of the latter is either 255 or 257.

Experiment 22.

Fix two monochord wires to produce two notes exactly in unison. Then move very slightly one of the bridges so that beats are produced. What is the effect of moving the bridge slightly farther in the same direction?

Beats may be produced very effectively by employing two tuning forks of the same pitch and slightly decreasing the frequency of one by loading one of its prongs.

This may be done by attaching a small strip of lead or even a bead of wax at the free end of one prong.

Exercises 6.

1. Give six sketches showing parts of mechanisms which are vibrating in equal intervals of time.

2. Give the periodic time in each of the above cases. If you do not know the exact periodic time give the average. Take the periodic time to be the time taken to complete one to and fro movement.

3. The vibration frequency may be defined as the number of periods in one second. State the average vibration frequency in Question 1.

4. Take a piece of wood of suitable length and of cross-section $\frac{3}{4}''$ by $\frac{1}{8}''$; fasten one end in a vice. Determine the period and frequency of the free end. Repeat the experiment, taking the same length and the cross-section $\frac{1}{2}''$ by $\frac{1}{8}''$. What can you deduce from this experiment ?

5. What do you understand by a vibratory motion ?

6. Describe the way in which sound is propagated through air.

7. Distinguish between longitudinal and transverse vibrations and illustrate your answer by sketches.

8. Define frequency, amplitude, and wave-length.

9. The following figures show the velocity of sound in feet per second. Express them in metres per second.

Velocity of sound in air, 1,080 ; in water, 4,900 ; in wet sand, 825 ; in granite, 1,664 ; in iron, 17,500 ; in copper, 10,378 ; in pine wood 11,000.

10. Boys are placed at two successive telephone poles and one of the poles is struck by a blow. Will the sound transmitted through the wood and wire be heard before the sound transmitted through the air ? Use the figures in the preceding example to give an approximate result.

11. Explain how you would perform a series of experiments on a monochord and show from the results of your experiments that the following statements are true :—

(1) The frequency is inversely proportional to the length of the string.

(2) The frequency is inversely proportional to the diameter of the string.

(3) The frequency is inversely proportional to the square root of the density of the material of which the string is made.

(4) The frequency is directly proportional to the square root of the tension by which the string is stretched.

12. Assuming that the middle C has a frequency of 256 vibrations per second, determine the note emitted when a card touches a wheel with 64 teeth revolving at the rate of 8 revolutions per second.

13. In a mechanism, one of the wheels has 32 teeth and is revolving 16 times per second. If the tip of the wheel rubs against a piece of material so that a musical note is produced, state what is the note? How many revolutions per second must the wheel make to produce the middle B?

14. The following table shows the vibrations per second of the sequence of the white notes on the piano commencing with the middle C :—

Notes	C	D	E	F	G	A	B	C′
Vibrations per second	256	288	320	341·3	384	426·6	480	512

What is the interval between each of the notes as compared with C? Express your results as vulgar fractions expressed in their simplest forms.

15. If the middle C has a frequency of 256 show the position on the treble musical stave of notes having the following frequencies :—320, 384, 512.

16. The humming noise which accompanies the flight of certain flies is caused by the beating of their wings. The following information is supplied :—The wings of a gnat make 50 per second beats, of a wasp, 110, and of the common house fly, 330. From your own experience state what is the approximate note in each case and explain how the figures supplied bear on your answer.

17. It has been noted that a series of taps will blend into a musical note when their number exceeds 20 per second. Using this information state which of the following machines will be likely to cause a musical note :—

Ingersoll " Eclipse Rock Drills "—

Size and type . . .	B2	C6	E3	FA
Strokes per minute . . .	500	375	350	300

Petrol engines—

Maximum strokes per minute of the valve tappets	725	925	625	1050

18. A wheel with 64 teeth revolves at 4 revolutions per second and a card just touches the wheel. What note is produced ? If some of the teeth are now broken off in an irregular manner, state how the sound produced will be altered.

19. What is the difference between a musical sound and a noise ?

20. A gas engine discharges its exhaust gases at a pressure of 25 lbs. per sq. in. above atmosphere into a long exhaust-pipe 65 feet long. Oscillations or waves of pressure are set up in the exhaust-pipe. If the engine exhausts 80 times per minute, give a sketch of the pressure wave and state what will be its frequency.

21. Discuss the use of the " silencer " on a motor cycle. Why is there a difference in the noise produced by the exhaust when the " cut-out " is used ?

CHAPTER VII

HARMONICS AND RESONANCE

Nodes and Loops.—When a length of wire is vibrating there are certain points which do not move at all and certain others which move a maximum amount. The former are called "nodes" and the latter "loops." In Fig. 33, which showed the simplest form of vibration, it is seen that we have nodes at the points where the wire crosses the bridges and a loop at a point mid-way between the bridges.

Whatever else happens it is obvious that nodes must occur at the points where the wire crosses the bridges, but they *may* occur elsewhere also. For example, it is quite possible by gently touching a wire midway between the bridges and bowing it (with a violin bow) at a point about one quarter of its length from one bridge, to induce it to vibrate in a manner shown in Fig. 37.

If such an effect is produced the note emitted is found to be an octave above that given by the simple vibration (which is called the "fundamental"). In other words, the frequency of this note is exactly double that of the fundamental.

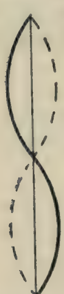

FIG. 37.

It is possible to produce a vibration in the wire similar to that shown in Fig. 38, in which there are *three* additional nodes. It is found, however, that the conditions for this vibration are much more difficult to obtain than those in the previous case. It is readily seen that other vibrations would be *possible* if the conditions for producing them could be obtained.

FIG. 38.

Now it has been shown that a stretched wire can vibrate in several ways at once

and in fact always does so. The "note" which is heard is usually the fundamental, but more complex vibrations are taking place at the same time, although the amplitude of these is usually very small indeed compared with the amplitude of the fundamental. It is for this reason that we do not hear *as such* the notes produced by these small vibrations, but the sound which they produce goes to make up the *quality* or *timbre* of the note.

Harmonics.—These small complex vibrations which are nearly always present in a note are called "harmonics" or "overtones." The vibrations shown in Figs. 37 and 38 would be called the first and third harmonics (or more correctly the "harmonics of the second and fourth orders") respectively.

If "middle C" be sounded on a piano and the note is taken up and produced in exact unison by other instruments, such as a violin and a cornet, the note is still middle C, and as such its frequency is 256 vibrations per second. The three notes, however, are very far from being alike in quality.

This difference of quality is due to the presence of a different set of harmonics in each case. Students who play the violin will be familiar with the formation of harmonics. Normally a violinist presses a string down with his fingers and the bow sets up a vibration in the portion of the string between his finger and the violin bridge. This is a fundamental vibration. In such cases he obtains a note of higher frequency by actually reducing the length of the string which vibrates.

When, however, the violinist wishes to produce a harmonic, he just *touches* the string at a certain point and bows very gently and the *whole* string is set in vibration: that is, the vibration is not confined to the portion of the string between his finger and the bridge.

Resonance.—If a very long and very heavy pendulum is erected, it requires a considerable effort to set it in motion. If, however, it is found either by trial or calculation that the period of such a pendulum is, say, 5 seconds, it is possible to set the pendulum swinging by applying to the bob a comparatively small force at regular intervals of 5 seconds.

The effect of any *one* of these impacts is, of course, negligible, but owing to the regularity of their application, their cumulative effect is considerable. If after a while the forces are applied at the wrong moment it is possible to counteract the effects of those previously applied.

Experiment 23.

Erect a heavy pendulum and determine its period in the usual way. Having allowed the pendulum to come to rest, apply very small taps with one finger, at intervals exactly equal to the period. Note the growth of the amplitude of the pendulum. Try the effect of a few ill-timed taps.

Experiment 24.

Arrange two monochord wires to give notes exactly in unison. If one wire is now plucked, and, after the note has sounded for one or two seconds, its vibration is stopped, it will be found that the second wire has taken up the vibration and is now emitting a note.

This phenomenon is called "resonance" and its explanation lies in the principle involved in Experiment 23. In this case we have a wire vibrating and sending out in all directions waves of sound of a given frequency. These waves beat up against the other wire, and although one wave is insufficient to make any appreciable difference, their cumulative effect is very considerable, as in the case of the small forces applied to the heavy pendulum.

The frequency with which a body freely vibrates is called the "natural" frequency, and when impulses reach it having a period equal to that of the natural frequency the vibrations are said to "synchronise," and when these conditions are fulfilled resonance usually takes place.

Experiment 25.

Examine the "action" of a piano. It will be seen that the striker which sets the wire in vibration when a note on the keyboard is depressed withdraws from the wire, leaving it free to go on vibrating. As soon as pressure is removed from the key, however, a damper is placed upon the wire and the vibration immediately ceases.

If a key is depressed very slowly the striker fails to reach the wire and no note is emitted, but so long as the key is kept down the damper is raised from the wire.

Depress middle C in this way, and while it is held down strike the C below it, hold it down for a second or two, and then release it. Middle C will now be sounding.

Here we have a case in which the first harmonic of the lower C (which is equal in frequency to middle C) has synchronised with the fundamental of middle C and set the wire in vibration.

Again, having silently depressed middle C, strike the C above it, allow it to sound for a second or two, and release it. It will be found that middle C is now emitting a note of an octave higher. In this case the vibrations of the fundamental of upper C synchronised with the first harmonic of middle C and hence set the wire in vibration of the form shown in Fig. 37.

Note.—The student will find a certain amount of mathematical treatment of harmonics in the authors' *Second Course in Mathematics for Technical Students*, chapter x. Also the author's paper on "Vibration of Spars in Aircraft," published in *Engineering*, vol. CIX., page 201, gives some practical applications of harmonics.

Exercises 7.

1. Show by the aid of sketches what you understand by the terms, "node" and "loop."

2. Show by the aid of a sketch a wire vibrating in such a manner as to sound its fundamental note. What difference in the sound is caused by the wire vibrating in 2, 3, or 4 separate portions respectively ?

3. The same note is sounded on a piano and a violin. Explain why it is easy to distinguish the two types of instruments although the same note has been sounded in each case.

4. What do you understand by the following statement :— In the case of vibrating strings the frequencies of the suc-

cessive overtones are as the series of the natural numbers. Give drawings to illustrate your answer.

5. Experiment 21 gives the following formula :—

$$N = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

The density of the following materials, in grammes per cubic centimetre, is as follows :—Aluminium 2·58, copper 8·93, mild steel 7·68, wrought iron 7·8, cast steel 7·9. You are given wires made of these materials of the same diameter and length and loaded with the same weight. An experiment is then performed on the monochord with each of these wires ; arrange the metals in sequence, placing the wire which gives the highest note at the top. Show that experiment and calculation give the same result.

6. A common shop method of distinguishing bars of metal is to listen to the note sounded when the bar is struck. If equal bars of mild steel, wrought iron and cast steel are provided state how you would distinguish them by this method.

7. In any note sounded by an open organ pipe the harmonics may have waves of frequencies of $2n$, $3n$, $4n$, etc., where n is the frequency. In a closed pipe only frequencies of $3n$, $5n$, etc., exist. The lack of even harmonics in the latter case gives a nasal quality to the tone. Show diagrammatically the harmonics in the two types of pipes.

8. In preparing the foundations of engines it is very necessary to arrange that any vibrations arising from want of balance should not be transmitted to the rest of the building. One method adopted is first to fix concrete foundations for the engine, then to place a layer of compressed felt between bed plate of the engine and the concrete face. Give reasons to show why the felt should help to prevent the transmission of vibration.

9. Show by the aid of sketches how vibration is produced in the following cases :—(*a*) by blowing sharply across the end of the hole in a key, (*b*) in an ordinary tin whistle.

10. When two notes are not quite in tune the resulting sound is found to alternate between a maximum and a minimum of loudness which recurs periodically. Show by the aid of a diagram how this is caused.

11. An aeroplane with two engines is heard some distance away and the sound is heard as a succession of beats. It is known that the engines are not making exactly the same number of revolutions per minute, but the average number of revolutions is 2,000 per minute. Give explanations why the sound is heard in beats.

CHAPTER VIII

VIBRATION OF REEDS AND GAS COLUMNS

Vibration of Reeds.—So far we have only considered the vibration of a stretched string (or wire) and it has been assumed that the material has been flexible and tension has been necessary before vibration became possible.

By a "reed" is meant a strip of material which possesses the property of "stiffness." This enables vibration to take place without the material being in tension.

A reed must always be fixed at one end, and this ensures the formation of a node at this point. Whatever form the vibration takes there must also be a "loop" at the free end of the reed.

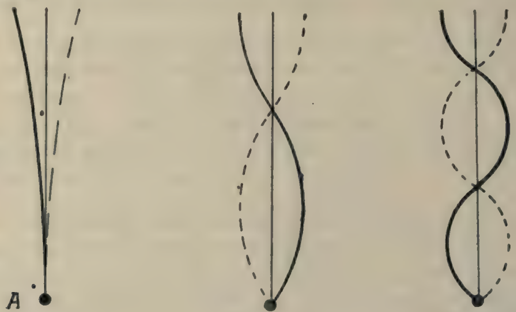

FIG. 39. FIG. 40. FIG. 41.

Fig. 39 shows the simplest form of vibration of a reed. Figs. 40 and 41 show the forms of the vibrations corresponding with the first and second harmonics respectively.

Relative Frequency of Harmonics.—We have already seen that the pitch of a note is dependent upon its frequency,

and this in turn is inversely proportional to the length of the sound-wave. Now, although the wave-length of the sound in the air is usually quite different from the length of the wave in the vibrating body, the two lengths are directly proportional.

We will now examine Figs. 33 and 37 to 41, and find the relative value of the frequency (n) in terms of the length of the string or reed (L).

			Wave-length. λ	Relative Frequency. $n = \frac{1}{\lambda}$
Fig. 33	Stretched String	Fundamental	$\frac{2L}{1}$	$\frac{1}{2L}$
Fig. 37	Stretched String	1st Harmonic	$\frac{2L}{2}$	$\frac{2}{2L}$
Fig. 38	Stretched String	3rd Harmonic	$\frac{2L}{4}$	$\frac{4}{2L}$
Fig. 39	Reed	Fundamental	$\frac{4L}{1}$	$\frac{1}{4L}$
Fig. 40	Reed	1st Harmonic	$\frac{4L}{3}$	$\frac{3}{4L}$
Fig. 41	Reed	2nd Harmonic	$\frac{4L}{5}$	$\frac{5}{4L}$

It must be clearly understood that these values of frequency are *relative* and the assumption is made that all conditions remain constant. The *actual* value of the frequency depends upon the length, weight, and tension of the string, or the length, weight, and stiffness of the reed.

Suppose the conditions were such that the fundamental note in each case was middle C. The first harmonic of the stretched string, having a frequency double that of the

fundamental, would be the C above, usually written C′. The third harmonic has a frequency double that of C′, which takes us up another octave to C″.

Fig. 42 shows the form of vibration of the second harmonic of a stretched string, and we see that $\lambda = \frac{2L}{3}$ and $n = \frac{3}{2L}$. Hence, if $\frac{1}{2L} = 24$, $\frac{3}{2L} = 72$, which a reference to Fig. 35 will show is the note G′.

Considering the harmonics of the reed in the same way we may make a table as follows :—

FIG. 42.

	Stretched String.		Reed.	
	Relative Frequency.	Note.	Relative Frequency.	Note.
Fundamental . .	$\frac{1}{2L}$	C	$\frac{1}{4L}$	C
1st Harmonic . .	$\frac{2}{2L}$	C′	$\frac{3}{4L}$	G′
2nd Harmonic . .	$\frac{3}{2L}$	G′	$\frac{5}{4L}$	E''
3rd Harmonic . .	$\frac{4}{2L}$	C''	$\frac{7}{4L}$	A''
4th Harmonic . .	$\frac{5}{2L}$	E''	$\frac{9}{4L}$	D'''

Bearing in mind that the "notes" given above refer to the harmonics of a stretched string and a reed, both emitting middle C as the fundamental, we see that the harmonics are quite different, and this accounts for the difference of quality or timbre between the notes emitted by these two means.

Higher harmonics are of course produced, but sufficient are shown in the table to enable the student to extend the

series if he wishes. The designer of musical instruments has to study which harmonics give a pleasant and which an unpleasant quality to a note and he then has to foster the former and eliminate the latter by suitable design in the instrument.

Experiment 26.

Obtain a strip of steel (or a steel knitting-needle) and fix it in a vice so that the free portion will emit a note when made to vibrate. Make a table showing the relation between the length of the reed which emits any note and the length which emits the octave of that note. (Remember that if N is the frequency of any note the octave above has a frequency of 2 N and the octave below a frequency of $\frac{1}{2}$ N.)

Tuning Forks.—The vibration of a tuning fork is only a special case of the vibration of a reed. It may be regarded as two reeds, attached at one end. Fig. 43 shows the form of vibration. The two prongs alternately approach and recede from each other.

FIG. 43.

A fork of this nature has two advantages over a single reed. Its note is sustained: that is, the vibration continues much longer than it would in a single reed, and unless the vibration is violent the note emitted is singularly "pure." In other words, harmonics are almost absent.

Tuning forks of standard pitch may be obtained and these form a convenient standard of reference. Forks may be set in vibration by gently striking, but a better method is to draw a violin bow across one prong. If a fork is placed with its base on a piece of wood, or better still a wooden box, the vibrations of the fork are conveyed to the wood and the volume of sound emitted is greatly increased.

The Vibration of a Column of Gas.—So far we have only considered the vibration of solid matter, and its power to vibrate has been due to its possessing the property of elasticity. An elastic body is one which, when deformed by

an external force, has the power of restoration to its normal shape as soon as the force is removed.

When the properties of gases were being investigated we saw that the volume of a given mass of gas varied inversely as the pressure. Now, as the gas is compressed by an external force and recovers its original volume when that force is removed we see that gases are "elastic." That being so, they should be capable of vibration like other elastic bodies.

Experiment 27.

Arrange eight test-tubes in a stand. On blowing sharply across the mouth of any tube a note is emitted. This is due to the vibration of the column of air within the tube. If a little water is placed in the tube it reduces the length of the column of air and the pitch of the note is raised.

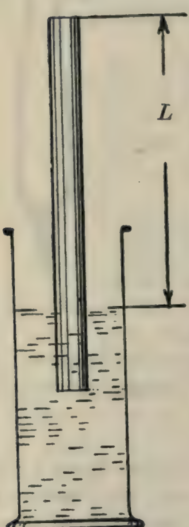

FIG. 44.

Commencing with the first tube empty place water in the remainder to such a depth that the musical scale is obtained from the eight tubes. Now measure the length of each tube above the level of the water and plot a graph showing the relation between this length and the relative frequency of the note emitted. (The relative frequencies of an octave are given in Fig. 35.)

Experiment 28.

Obtain a tall cylinder and nearly fill it with water. Support a glass tube, open at both ends and about 1 inch in diameter, with one end below the surface of the water, as shown in Fig. 44. Now, having set a tuning fork in vibration, hold it over the open end of the tube and raise or lower the tube in the water until resonance is produced and the tube is emitting a note of similar pitch to that of the fork. Measure the length (L) of the vibrating column of gas.

If forks of other frequencies are available the experiment should be repeated, and a graph plotted showing the relation between length of column and frequency.

Now let us consider in detail what is happening in this experiment when resonance occurs. Suppose at a particular moment the lower prong of the tuning fork is moving downwards. This impact on the adjacent air particles will start a compression wave down the tube. This wave will be reflected at the surface of the water and return up the tube. For resonance to take place the wave must reach the lower prong of the fork as it moves upwards.

If the frequency of the fork be N vibrations per second it will be seen that the interval between the prong moving downwards and its commencing to move upwards is $\frac{1}{2N}$ seconds. In this interval the sound wave has moved down the tube and up again, a distance of $2L$.

$$\text{Now velocity} = \frac{\text{space}}{\text{time.}}$$

$$\therefore \text{Velocity of sound} = \frac{2L}{\frac{1}{2N.}}$$
$$= 4LN.$$

This method of determining the velocity of sound is not very accurate. A certain amount of disturbance takes place at the open end of the tube. A closer approximation is given by :—

$$\text{Velocity} = 4N(L + 0{\cdot}4d).$$

where d=diameter of tube.

The student should obtain a value for the velocity of sound by this method. It should not be necessary to add that all measurements of length should be made in the same unit : say the foot or the centimetre.

The student should note that the column of gas producing the sound in the preceding experiment was vibrating with a *longitudinal* vibration, and not *transversely* as in the case of a stretched string or a reed. It is convenient neverthe-

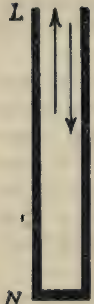

FIG. 45.

less to speak of nodes and loops. In the case under consideration there was a node at the closed end of the tube (*i.e.* at the water surface) and a loop at the open end. This is represented in Fig. 45, the letters N and L representing node and loop respectively. The arrows represent the direction of motion of the air particles.

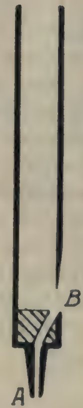

FIG. 46.

Organ Pipes.—Fig. 46 shows a section of a simple organ pipe made of wood. Air passes from the wind chamber through the inlet A and passing up the channel shown, impinges on the lip B. This causes the column of air within the pipe to vibrate, but unlike the column of air in the tube in Experiment 28 this pipe is open at both ends and there must therefore be a loop at both ends of the pipe. The form of vibration for the fundamental note is shown in Fig. 47. The forms of vibration for the 1st and 2nd harmonics are shown in Figs. 48 and 49 respectively, and it will be noted that there is a loop at both ends of the pipe in all cases.

Closed Pipes.—The organ pipe just considered is known as an "open pipe." On a modern organ, however, there are

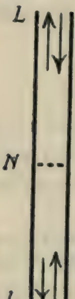

FIG. 47.

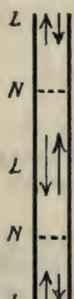

FIG. 48.

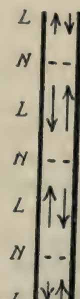

FIG. 49.

FIG. 50.

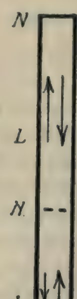

FIG. 51.

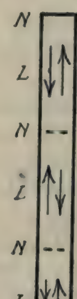

FIG. 52.

always a certain number of pipes exactly similar to Fig. 46, but having a tightly-fitting stopper driven into the top. These are known as stopped or closed pipes.

The effect on the nature of the vibration is to make a node at the closed end of the pipe. The form of vibration for the fundamental is shown in Fig. 50, while Figs. 51 and 52 show the forms of vibrations of the first and second harmonics.

Letting L represent the length of the pipe we may now compile a table similar to that referring to the vibration of stretched strings and reeds.

			Wave Length. λ	Relative frequency. $N=\frac{1}{\lambda}$
Fig. 47	Open organ pipes	Fundamental	$\frac{2L}{1}$	$\frac{1}{2L}$
Fig. 48	Open organ pipes	1st Harmonic	$\frac{2L}{2}$	$\frac{2}{2L}$
Fig. 49	Open organ pipes	2nd Harmonic	$\frac{2L}{3}$	$\frac{3}{2L}$
Fig. 50	Closed organ pipes	Fundamental	$\frac{4L}{1}$	$\frac{1}{4L}$
Fig. 51	Closed organ pipes	1st Harmonic	$\frac{4L}{3}$	$\frac{3}{4L}$
Fig. 52	Closed organ pipes	2nd Harmonic	$\frac{4L}{5}$	$\frac{5}{4L}$

It will be observed that the frequency of a closed pipe is exactly half that of an open pipe of the same length, that is, the note emitted is an octave lower. On an organ an 8 ft. " closed " pipe gives a note of the same pitch as a 16 ft. " open " pipe. It will be seen from the table, however,

that the harmonics differ, and hence the note, although of the same *pitch*, is of quite different *quality*, as every organist knows.

Exercises 8.

1. A piece of thin wood was fixed in the vice and set in vibration. This experiment was repeated 4 times for the following lengths :—2′ 0″, 1′ 6″, 1′ 0″, 6″.

If the frequency $n=\frac{1}{4l}C$,

where l=the length, and C equals a constant, determine the frequency ratio between the 2′-0″ and the other three bars.

2. Give a sectional sketch of a mouth-organ and mark clearly the vibrating tongue.

3. Examine, if possible, an oboe, a bassoon, and a clarinet. Give sectional sketches showing diagrammatically the air current and the reed.

4. If the vibration number of the lowest note on the piano, A_4, is 27, and the speed of sound in air is 1,130 feet per second, find the wave-length of the note as it travels through the air.

5. A thunderclap was heard $4\frac{1}{2}$ seconds after the accompanying lightning-flash was seen. How far away did the flash occur ?

6. On a still day the human voice may be heard for 150 yds. and rifle fire for 5,300 yds. Assuming the usual velocity of sound in air state what time sound will take in travelling the distances given.

7. A flat disc has 30 holes, spaced equally, with the centres of the holes on the same circle. If the disc is rotated rapidly and a jet of air is blown through a small glass tube and made to impinge on these holes, state what will happen :—

(*a*) When the disc runs at a uniform speed.
(*b*) A variable speed.
(*c*) Double the speed in (*a*).

8. In the above question state how many revolutions per minute the disc must make to give the following notes of standard musical pitch :—

Note	C′	D′	E′	C″
Frequency per second . .	261	293	328·9	522

9. Explain why the sirens used in steamers and manufacturing works often give sounds which vary between a low note and a shrill shriek.

10. Explain, by the aid of sketches, why musical sounds may be produced on a tin whistle and not when air is blown through a parallel piece of tubing of the same diameter and length as the whistle.

11. Describe an experimental method for obtaining the number of vibrations per second made by a tuning fork.

12. Explain why the pitch of the sound rises as water is poured into a deep vessel.

13. On starting an electric motor it will be noted that the pitch of the sound gradually rises. Why is this ?

14. Make a sketch of a bugle. Mark off the length of the air column. Explain why this instrument, which has a fixed length, can produce several notes.

15. Make a sectional sketch of the mechanism for the formation of sounds in a gramophone.

16. The velocity of sound in air (V) depends on the temperature and may be calculated from the formula :— $V = 330\sqrt{1 + 0{\cdot}004t}$. Where V is measured in metres per sec., t is the temperature in degrees Centigrade. Calculate V for 10°, 25° and 30° C.

SECTION IV.—LIGHT

CHAPTER IX

PHOTOMETRY

Propagation of Light.—It is common experience that a body exists in the direction in which we "see" it, that is, in the direction of the rays of light which pass from the body to our eye. From this it follows that light travels in straight lines.

Light travels by means of a wave motion, and in this respect is similar to sound, but the wave motion which transmits light differs in three important particulars from that by which sound is conveyed.

(1) It is a transverse wave motion, whereas sound waves are longitudinal.

(2) It can traverse a vacuum, which sound waves cannot do.

(3) The waves are very much shorter, and the motion very much quicker than is the case with sound.

The form of transverse wave motion was shown in Fig. 31. The wave length of light depends upon its colour, red light having the longest and violet the shortest wave. Yellow light, which is of medium wave-length, has a wave which is rather less than 0·00006 cms. long. In other words, there are about 43,000 complete waves of yellow light in one inch. It has already been mentioned that light has a velocity of about 186,600 miles per second.

We are able to see the sun, moon, and stars by means of the light which proceeds from them. We know, however, that the earth's atmosphere is confined to a comparatively shallow layer. Away out in space we have the absence of all forms of matter, that is, a vacuum. Yet light passes through it quite readily.

Hence, whatever it is that conveys the transverse wave

motion of light, it is *not* matter. Physicists call it "the ether."

Shadows.—Since light travels in straight lines, it cannot pass round a large obstacle as can sound. This accounts for the formation of shadows. It is generally found that the shorter the wave-length of a vibration the greater the difficulty it has in passing an obstacle.

Consider a source of light which is a point, as S in Fig. 53 (an electric arc or a lime-light nearly fulfils this condition). Light passes from this point in straight lines, but if an opaque sphere, as AB, be placed in the path of the rays, a conical space will exist behind the sphere, into which no ray of light from S can pass. If a screen is provided, the absence of light in this space will be denoted by a shadow CD.

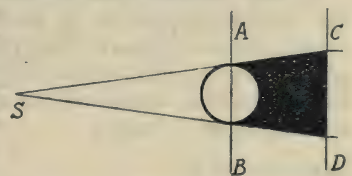

FIG. 53.

If the source of light is a *point*, as we have supposed, or at any rate is very small compared with the size of the opaque object, the shadow will be very well defined, that is, the boundary between the illuminated part of the screen and the shadow will form a sharp contrast. If, however, the source of light has an appreciable size (as is usually the case), the shadow is very badly defined, there being a fringe of partial illumination around the shadow itself.

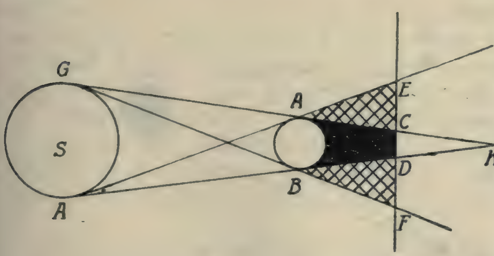

FIG. 54.

Consider the case shown in Fig. 54. S is the source of light, and AB the opaque object as before. The large illuminant may be regarded as being made up of a number of

points. Let us think of two extreme cases, namely, the points G and H.

Rays of light passing from G will fall on AB in such a way as to cast a shadow on the screen at CF, while rays from H will fail to reach the portion of the screen DE. Since these are the extreme cases for the illuminant, they must be the extreme cases for the shadows also.

Now it is easily seen that the portion of the screen CD is in shadow for all cases, the portions CE and DF are in shadow for some cases only, while the screen outside EF is not in shadow at all.

The dark portion of the shadow (CD) is called the *umbra*, while the fringe which diminishes in intensity toward the edge is called the *penumbra*.

It will be seen that if the source of light is larger than the opaque object (as is the case in Fig. 54) the umbra is formed by a cone converging to a point at K. If the screen is placed beyond this point, there is no umbra shown at all, but a nebulous shadow in the form of a ring. This is called an "annular" shadow.

Experiment 29.

The Photometer.—Obtain an opaque rod about an inch in diameter, and fix it in a vertical position with a small screen of white paper or other suitable material a few inches behind it.

If now a small light (for example, a candle) be placed in front of the rod, a shadow is formed on the screen. If a second light (say, a small electric bulb) be placed in front of the rod, but not in line with the candle, another shadow is produced.

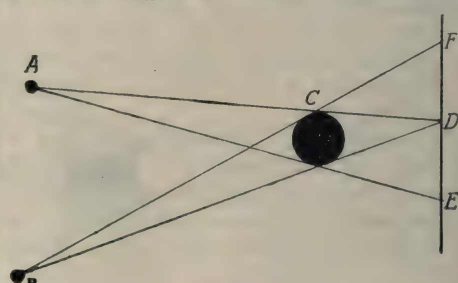

FIG. 55.

If the lights are placed as A and B

in Fig. 55, it is possible to have the two shadows side by side and just in contact. It is clear from this diagram that the part of the screen represented by DF is illuminated by the light A only, while the portion DE receives light only from B.

Hence if the relative position of the lights is so adjusted that the two shadows appear of equal intensity, it is obvious that the screen is receiving an equal illumination from both lights.

We can use this fact to compare the relative intensities of the two illuminants.

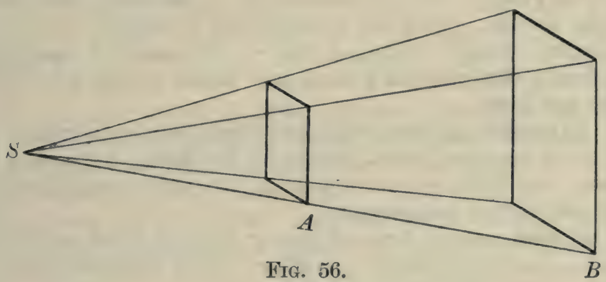

FIG. 56.

First let us consider Fig. 56, in which S is a point of light from which rays are passing in all directions. If a square screen is placed at A, the amount of light which illuminates the surface of the screen is conveyed by the rays passing within the pyramid formed by joining the point S to the four corners of the screen.

If a screen were placed at B instead of A, it is easy to see that the same amount of light would now fall on a screen of much larger dimensions. If the distance SB is double that of SA the side of the square B will be double that of A. That is, the area of B will be four times as great as the area of A, and since the same amount of light falls on both screens, it is clear that the illumination of B will be only a quarter as bright as that of A.

Law of Inverse Squares.—From the foregoing it is clear that, other things being equal, the intensity of illumination

of a screen varies inversely as the square of the distance from the illuminant to the screen.

Thus in Fig. 55, if P_1 is the power of the light at A and P_2 that at B, and the distance from the screen to A and B is respectively D_1 and D_2, we have :—

$$\frac{P_1}{P_2} = \frac{D_1^2}{D_2^2}$$

Candle-Power.—The common unit of intensity of a source of light is that given by a candle of a certain type. The "standard candle," as it is called, is made of spermaceti wax, has a diameter of $\frac{7}{8}$ inch and burns at the rate of 120 grains per hour.

As a matter of fact this standard has now fallen into disuse for accurate work owing to its being somewhat uncertain, the light given depending to some extent on the condition of the atmosphere.

In modern work a form of burner is used which is fed with pentane (a very volatile paraffin). The Harcourt pentane burner gives 10 candle-power, and this is now used as the international standard.

Ex. 16. In Fig. 55 let the source of light at A be a standard candle, and that at B be of unknown power. The distance from A to the screen is 56 cms., and from B to the screen 85 cms. Find the candle-power of B.

$$\frac{\text{Candle-power of } B}{1} = \frac{85^2}{56^2}$$

$$\begin{aligned}\text{Candle-power of } B &= \left(\frac{85}{56}\right)^2 \\ &= (1{\cdot}517)^2 \\ &= 2{\cdot}3\end{aligned}$$

The advantage of this type of photometer is that it is not necessary to work in an entirely dark room. The presence of another light in the room will not affect the result, provided that the additional shadow produced does not overlap

the two which are being compared. A disadvantage of the method lies in the difficulty experienced when the sources of light are not approximately points.

Experiment 30.

The Grease-Spot Photometer.—Make a small screen of white blotting-paper and drop a spot of oil on it. Erect it in a darkened room, and place a light on one side of it.

Viewed from the side on which the light is placed the oiled spot appears dark on a white ground. This is due to the fact that, the spot being translucent, the light passes through it, while it is reflected from the rest of the paper.

Viewed from the other side, however, the spot appears bright on a dark ground. Assuming that *all* the light falling on the unoiled screen is reflected, and that that falling on the oiled spot is wholly diffused, it is clear that the spot should appear as bright in the second case as the remainder of the screen did in the first case.

Now place another light on the side of the screen opposite to the first light, adjusting the second light until the grease-spot is not distinguishable from the rest of the screen. Since the illumination must now be equal on both sides of the screen it is possible to compare illuminating powers as before.

If this method were perfect, the spot, having been made indistinguishable from the remainder of the screen when viewed from one side, should appear so when viewed from the other side. Such, however, is seldom the case. This is due to the fact that the light is more scattered by reflection from the white paper than it is by diffusion through the grease-spot.

In practice, therefore, one should aim, not at getting the grease spot to disappear, but producing a like contrast between spot and screen when viewed from both sides.

Exercises 9.

1. Light has a velocity of about 186,600 miles per second. What is the velocity in centimetres per second ?

2. Give simple experiments which prove that light travels in straight lines.

3. The following wave-lengths in "tenth metres" (10^{-10} metres) of characteristic bands and lines in the solar spectrum, are given :—4861·496, hydrogen. 4307·9, calcium. 5167·5, magnesium. 3820·56, iron. Express these wave-lengths as decimals of a metre.

4. If the wave-length is expressed in "seventh metres" (10^{-7} metres), express the following wave lengths in decimals of a centimetre :—4·86, 8, 20, and 25.

5. Give a list of materials grouped under the following headings :—opaque, translucent, and transparent.

6. If it is approximately 93,000,000 miles from the earth to the sun, determine how long light will be in travelling from the sun to the earth.

7. A candle is viewed through a pin-hole camera. Explain why the image is inverted.

8. A point source of light is S (Fig. 53), and a cardboard disc AB, 2″ diameter, is placed with its vertical diameter 6″ from S with (*a*) the surface of the disc parallel to the wall CD, (*b*) with the edge of the disc as shown in the figure. If the horizontal distance between B and D is 3″, determine graphically the shape and size of the shadow cast on CD in the two cases.

9. By the aid of sketches show how transverse wave motion differs from longitudinal wave motion.

10. Give some common illustrations to prove that light travels faster than sound.

11. A candle flame $\frac{1}{4}''$ high is placed 4″ from a pin-hole in a screen. Find the size of the image produced on screens which are placed at distances of 3″ and 8″ respectively from the pin hole.

12. Draw up a table, giving your impression of the sensitive sensation of brightness of the following :—A piece of white-hot iron, a carbon filament, a metallic filament and a gas-filled metallic filament lamp, an acetylene lamp, an oil lamp, a frosted incandescent electric lamp. The light giving the

impression of the highest brilliancy should be placed at the top of the table.

13. A ball 1″ in diameter is held 2″ from a luminous point of light. Give a drawing showing the shape of the shadow which is cast on a wall $1\frac{1}{2}$″ from a vertical diameter of the ball. Will there be a penumbra ?

14. What do you understand by the "law of inverse squares" as applied to photometry ?

15. The Hefner lamp is used as the German standard of candle power and the Hefner standard is equal to 0·9 International Candles. A test was carried out on Osram lamps and the following results represent Hefner units of candle power :—36·3 29·7, 1·0, 22·7, 30·7. What will be the value of these results in International Candle Power ?

16. Experimental observation shows that a white surface illuminated by one candle at a distance of one foot appears equally bright if illuminated by four candles at a distance of 2 ft., or nine candles at a distance of 3 ft. State what law you would deduce from these observations.

17. The intensity of illumination produced by the Standard Candle at the distance of one foot has been adopted as a practical unit of intensity of illumination and is called the "*foot candle*." How many "*candle metres*" are equal to one foot candle ?

18. Intrinsic brilliancy is a term used to express the degree of brightness of a light and is usually given in candle power per square inch of the luminous area. Molesworth gives the following values :—Carbon filament lamp, 375 c.p. per sq. in. ; metallic filament lamp (vacuum), 800 to 1000 c.p. per sq. in. ; metallic filament lamp, gas filled, 3,500 c.p. per sq. in. ; acetylene lamp, 100 ; oil flame, 3·8 ; gas flame, 2·5. What reasons are there for defining Intrinsic Brilliancy ? Contrast these results with your answer to Question 12.

CHAPTER X

REFLECTION

Laws of Reflection.—No one can witness a game of billiards without coming to the conclusion that in rebounding from the raised edge of the table the ball obeys definite laws. These laws may be called the laws of reflection, and they apply equally well to a beam of light falling on a mirror and a smoothly rolling billiard ball.

Mirrors.—For light to be reflected under conditions rendering measurement possible, it is necessary for the reflecting surface to be smooth, and free from irregularities. A piece of thin plate glass carrying a deposit of silver forms a suitable mirror. The surface of the silver next to the glass is the brighter, but it has the disadvantage of having the glass in front of it. For many purposes this disadvantage is not serious, especially if the glass is not very thick.

A beam of sunlight passing through a chink in a blind is readily reflected from a mirror, and the approximate direction of the reflected beam is indicated by the position of the spot of light which the reflected rays cast on the wall. This phenomenon is not readily adapted to measurement, but it is possible to get therefrom a general idea of the laws involved.

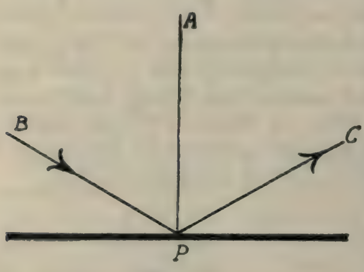

Fig. 57.

In Fig. 57 the horizontal line represents the reflecting surface of a mirror. A beam of light travelling in the direction BP meets the mirror at P, and the light is deviated in the direction PC.

P is called the "point of incidence," and a line BP passing along the direction of the beam represents the path of the "incident ray," while the line PC represents that of the "reflected ray."

A line drawn from the point of incidence at right angles angles to the surface of the mirror (as PA) is called the "normal."

The angle between the incident ray and the normal (the angle BPA) is called the "angle of incidence," while the angle between the reflected ray and the normal (the angle CPA) is called the "angle of reflection."

We are now in a position to state the laws of reflection.

The Laws of Reflection.—(1) The incident ray, the normal, and the reflected ray lie in the same plane.

(2) The angle of incidence and the angle of reflection are equal.

The following experiment should make this clear.

Experiment 31.

Obtain a strip of mirror about one inch wide and two or three inches long, a few pins and a sheet of paper. Fasten the paper to a drawing-board and by means of any suitable support cause the mirror to rest upon its edge with its reflecting surface at right angles to the surface of the paper.

Place a pin vertically in the drawing-board and close to the surface of the mirror (as F in Fig. 58, in which AB represents the mirror). Now place another pin about two inches from the mirror, and to one side of the first pin (as C). If now the eye be placed in a position such as D, the reflection of the rays of light passing from the pin C will cause an image of the pin to be seen (apparently behind the mirror) in the direction DE.

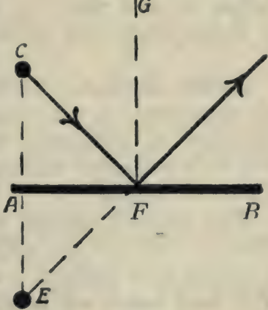

Fig. 58.

Place a pin near the eye (at D)

so that it, the pin at F, and the image appear in a straight line. By drawing a pencil-line along the edge of the mirror, and joining the points CF and FD, we have a record of the path of the incident ray and the reflected ray.

Draw the normal FG at right angles to AB, and measure the angle of incidence (CFG) and the angle of reflection (DFG). Repeat the experiment for several positions of C.

Experiment 32.

Fix a mirror and the pins F and G (Fig. 58) as in Experiment 31. Place the eye at D. Now take a long pin and place it behind the mirror so that the top of it (which should be seen above the mirror) is in line with the pin F and the image of C. Move the eye from side to side a few times, and if the top of the long pin gets out of alignment when the eye is moved, the pin should be moved until no such separation occurs.

When this condition is secured, we know that the long pin is not only in the *direction* of the image of C, but is actually occupying the *position* of that image, as E in Fig. 58. Join EF and CE.

By measurement demonstrate that the distance from D to the image at E is equal to the distance from D to the pin C measured along the path which the light has actually traversed, namely, CF and FD. Again, show that the image E is always as far behind the mirror as the pin C is before it, and that the line joining the pin C and its image E is perpendicular to the surface of the mirror.

Curved Mirrors.—The two laws of reflection given above are applicable to all cases, but so far we have only considered the case of reflection from a plane (*i.e.* flat) mirror. We will now consider one or two simple cases of reflection from a curved mirror.

Any curved surface may be regarded as composed of an infinite number of very small flat surfaces. Therefore, if a ray of light falls on a curved mirror, it behaves as it would if it had fallen on a plane mirror which was tangential to the curve at the point of incidence.

Concave and Convex Mirrors.—Most of the curved mirrors with which the student will deal at this stage of his work will be portions of the surfaces of spheres, and may be called spherical mirrors. If the reflecting surface is on the *inside* of the sphere, the mirror is said to be concave; if the *outside* of the sphere is the reflecting surface we have a convex mirror.

The arc of a circle shown in Fig. 59 represents a section of the reflecting surface of a concave spherical mirror. O is the centre of the sphere. It is clear that if P is the point of incidence of a ray of light the behaviour of the ray will be such as one would obtain from a plane mirror which was

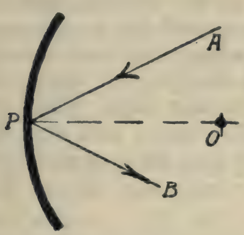

FIG. 59.

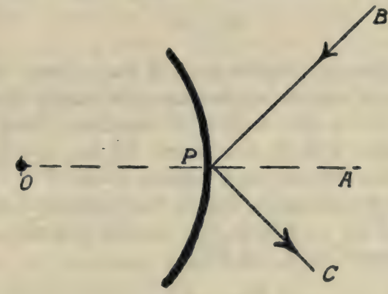

FIG. 60.

tangential to the sphere at the point P, that is, the radius OP will be the normal.

If AP is the incident ray the angle APO is the angle of incidence, and on making the angle OPB equal to this we have the angle of reflection and the path of the reflected ray PB.

Fig. 60 represents a convex spherical mirror. O is the centre of the sphere, and P is the point of incidence of the incident ray BP. This time we must produce the radius OP to A, to obtain the normal. The angles BPA and APC are respectively the angles of incidence and reflection, and are of course equal as before.

Experiments and further particulars referring to spherical mirrors will be given in Chapter XII.

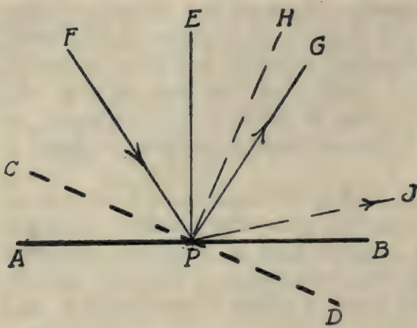

Fig. 61.

Effect of Rotating a Mirror.—Consider a mirror AB in Fig. 61 with a ray of light FP falling upon it. Since PE is the normal the reflected ray will pass along PG. Now suppose the mirror is turned through a small angle into a new position CD, the incident ray remaining stationary. PH will now be the normal, and the angle of incidence will have been increased by an amount equal to the angle through which the mirror has been turned.

Since the angles of incidence and reflection are equal, the latter will be increased by an equal amount also. Thus the reflected ray now passes along PJ. It is easily seen that the angle through which the reflected ray has been turned (viz. GPJ) is twice the angle through which the mirror was turned (viz. APC).

Experiment 33.

Repeat Experiment 31. Then rotate the mirror through about 20° without changing the position of the pins representing the incident ray. Record the path of the reflected ray as before and measure the angle through which the reflected ray has been deviated and compare it with the angle of rotation of the mirror. Repeat this experiment, using a number of different positions of the mirror. Tabulate your results.

Reflection from a Rough Surface.—If we place an electric light in front of a mirror the rays of light which, after reflection from the mirror, enter the eye actually form an image of the light itself. If the mirror is replaced by a piece of paper of the same size and shape, no image of the light is produced, but the paper is "illuminated."

Now the rays of light passing from the electric lamp to the paper are exactly similar to those which fell on the mirror, yet the visual effect is very different.

Owing to the smoothness of the reflecting surface of the mirror the reflected rays occupied a relationship to each other similar to that which existed before the mirror was reached, and hence they entered the eye under conditions very like those which would obtain if they had passed directly from the lamp to the eye.

The paper, on the other hand, has a surface very far removed from perfect smoothness, and although each individual ray obeys the laws of reflection, the angle of incidence at any particular point may be anything, depending upon the formation of the paper at that point. Hence the reflected rays have no relation to each other in any way resembling the initial conditions, and thus no image can be formed.

Exercises 10.

1. A ball is travelling in a straight line on a plane surface and comes in contact with a plane at right angles to the first surface. By the aid of diagrams trace the path of the ball under the following circumstances :—

(*a*) The ball strikes the surface normally and returns along the original path.

(*b*) The ball strikes the surface at an angle of 30° with the normal and returns along a path which makes 30° with the normal and 60° with the original path.

(*c*) As above, but the surface is struck at an angle of 60° with the normal and it returns along a path making 120° with the original direction.

2. A horizontal ray of light strikes a vertical mirror normally. Construct a diagram and state which is the Incident Ray, the Reflected Ray, the Normal, the angle of incidence and the angle of reflection.

3. Give a clear account of any experiment you have performed which proves that the angle of incidence is equal to the angle of reflection.

4. A horizontal ray of light comes from a point on a wall and strikes a vertical mirror so that the angle of incidence is 30°. Show, by means of a scale drawing, where the reflected ray will hit the wall if the distance of the wall from the mirror is 10 feet.

5. Taking the data as given in the above question, show by means of scale drawings where the light strikes the wall for the following cases :—

(*a*) Mirror and wall parallel, the light striking the mirror normally.

(*b*) The ray of light strikes the mirror normally; the mirror is kept vertical, but swung through 30° in a clockwise direction.

6. Measure the angle moved by the mirror in the above case and state whether the following statement is proved in this particular instance :—" When the mirror rotates through any angle, the reflected beam rotates through twice the angle."

7. A light wooden framework is built up in the form of an equilateral triangle ABC. Half-way down the side AB a small mirror is fixed at D with its plane parallel to the side AC. Pivoted at A is a framework $A' C'$, $C' B'$, resting on AC, CB, a mirror being fixed to the point A which moves as $A' C'$, $C' B'$ is moved. A horizontal ray of light EA strikes the mirror at A and is reflected to the mirror at D. State the inclination of the mirror at A to the base BC in order that the ray of light may leave parallel to the side BC.

In connection with this question the student is advised to examine if possible the construction of a sextant.

8. The curvature of a concave mirror is equal to one divided by the radius. Express as a decimal of an inch the curvature of the following concave mirrors :—Radius of the mirrors to be 12″, 8″, 200 millimetres and 80 millimetres, respectively.

9. Make scale drawings of the following concave mirrors of radius respectively 4·5″, 7·5″, 90 millimetres and 70 millimetres.

10. In each of the mirrors in Question 9 assume that a ray of light proceeds from a point $\frac{3}{4}''$ above and parallel to a radius. Draw in every case the ray and mark the angles of incidence and reflection. Measure the angles with a protractor and give their approximate values.

11. State the laws of reflection of light and explain with the aid of a diagram the formation of an image in a concave mirror.

12. In a lighting scheme for a drawing office, the following surfaces were fixed behind the half-watt lamps used for lighting: (*a*) Brightly polished metal reflectors. (*b*) Iron plates coated with aluminium paint on the underside. (*c*) *b* with a final coat of light dull-finished buff-coloured paint. In the latter case it was found that the favourable characteristics were freedom from glare and excellent diffusion. Give explanation of this.

13. Give an explanation as to why a beam of sunlight, admitted through a narrow slit to a darkened room, is rendered visible. What bearing has the following statement on your answer :—" In ordinary circumstances there are about 25,000 dust particles per cubic centimetre of air, rising to about 250,000 in the neighbourhood of large towns."

14. A ray of light makes an angle of 10° with the horizontal and strikes the mirror of a microscope, which is inclined at 25° to the horizontal. Will the light be reflected vertically ? If not, state the angle of the mirror necessary to accomplish this. Illustrate your answer by drawings.

15. What difference would you expect in the character of the light as reflected from (*a*) a conical white enamelled iron shade, 10″ lower dia., $1\frac{1}{8}$ hole at the top, and 5″ vertical depth. (*b*) As above, but made of opal glass.

16. Explain why light is reflected differently from plane and ground glass.

17. Why is a room with white walls much lighter than a similar room with dark green walls ?

18. A piece of shafting is firmly fixed at one end and supported freely at the other. The free end has an overhanging arm attached to it, just outside the bearing. Near each end of the shaft, small mirrors are attached and above each mirror a reading microscope with a scale is fixed. When the scale is illuminated the image of the scale can be seen by the microscope. A cross-hair indicates the position on the scale. Explain what will happen when the shaft is slightly twisted by means of weights attached to the overhanging arm.

Show approximately by means of sketches how the twist of the shaft is magnified.

19. Show how you would fix a pair of mirrors parallel to each other and one vertically above the other, in order to form a simple form of periscope.

20. Describe experiments by which you would demonstrate the two laws which show how a ray of light is reflected from a smooth surface.

21. Rays of light strike a horizontal plane mirror at the following angles :—10°, 15°, 30°, 45°, 60° and 75°. Show graphically the angle of deviation. Assume that the angles given are angles with the horizontal.

22. In the above question show how you would arrange a second mirror in order that the deviated ray may finally be deflected from the second mirror horizontally. Make a drawing in each case.

23. A building is surrounded by high walls. Give suggestions and sketches showing how you would get as much reflected light as possible into the rooms.

24. A number of rooms receive the whole of their light from a rectangular well, open to the air. It is noted that the walls of the well are made of white enamelled brick. Why is this ?

CHAPTER XI

REFRACTION

CONSIDER Fig. 62, in which AB represents a line of men marching with uniform speed. When the line reaches CD, it will be parallel to AB, and its direction of motion will be unchanged. Suppose, however, that in this position the extreme left-hand man is about to step into water about a foot deep. This will, of course, impede his progress, and he will consequently lag behind his fellows.

When the line reaches EF, half the men will have entered the water, but the man at F will have been in it longest, and will have lagged farthest behind. That is, the line will have been bent as shown, and although the two portions of the line will still be straight, the position representing the men in the water will indicate a change of direction in the forward motion.

We have already seen that sound does not travel with the same velocity in air and in water, and it is found that while a number of bodies are "transparent," that is, are capable of transmitting light, they do not permit the light to travel with the same velocity. Thus light travels more rapidly in vacuo than it does in air, and again, its velocity is greater in air than in glass.

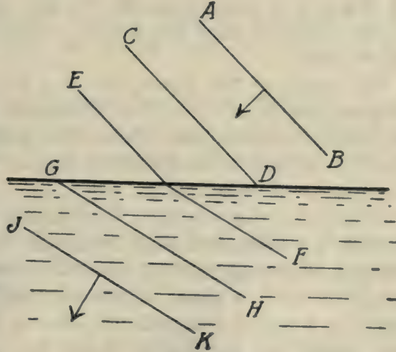

FIG. 62.

Hence, a wave-front of

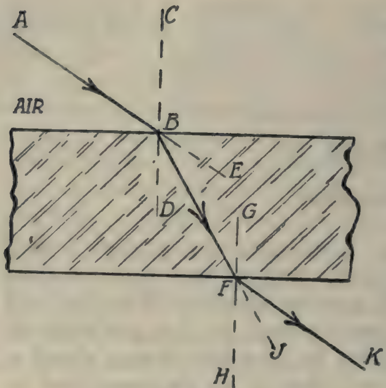

FIG. 63.

light passing obliquely from air into a block of glass will have its direction of motion changed, just as the line of men were diverted from their original path by their oblique entrance into the water.

Experiment 34.

Refraction.—Obtain a block of plate glass about $3'' \times 2'' \times 1''$, and place it with the large face on a horizontal sheet of paper. Place pins at A and B (Fig. 63), the latter being in contact with the glass. With the eye in the neighbourhood of K, fix two other pins F and K (the former being in contact with the glass) in such a way that all four pins appear in a straight line. By drawing a pencil line round the block of glass, and joining the points ABF and K, we have a representation of a ray of light passing obliquely through the glass.

Consider first the passage of a ray from air to glass. AB is the incident ray and B is the point of incidence. Through B draw CD perpendicular to the surface of the glass. This is the normal, and the angle ABC is the angle of incidence as before.

On entering the glass, the ray of light is deviated from its original path AB, in the direction BF. This phenomenon is called "refraction," and the angle between the path of the ray within the glass and the normal (viz. FBD) is called the angle of refraction.

When the ray reaches F it passes from glass to air, and it should be observed that here the deviation is *away* from the normal. The experiment should be repeated several times, commencing with a small angle of incidence, and increasing this angle so long as the path of the ray can be traced. Tabulate the angle of incidence, and the corresponding angles of refraction.

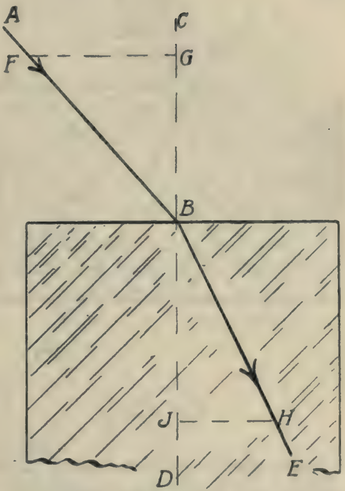

FIG. 64.

The Laws of Refraction.—A little thought should make it clear that the incident normal and refracted rays lie in the same plane.

We will now examine the relationship which exists between the angles of incidence and refraction. Consider any one case obtained in Experiment 34, and draw accurately the path of the ray as it passes from air to glass. Such a case is shown in Fig. 64.

From the point of incidence, B, mark off equal lengths BF and BH along the paths of the incident and refracted rays respectively. From the points F and H drop perpendiculars FG and HJ on to the normal. Measure these perpendiculars, and obtain the ratio $\frac{FG}{HJ}$.

Repeat this construction for every angle of incidence taken in the experiment. It will be found that the ratio is constant.

Students with a knowledge of the elements of trigonometry will readily see that the above-mentioned ratio is the same as the ratio of the sines of the angles of incidence and refraction.

The Refractive Index.—This ratio is called the refractive index of the glass or other medium and is generally denoted by the Greek letter μ (pronounced "mu"). We may say, therefore, that :—

$$\text{Refractive index} = \frac{\text{Sine of angle of incidence.}}{\text{Sine of angle of refraction.}}$$

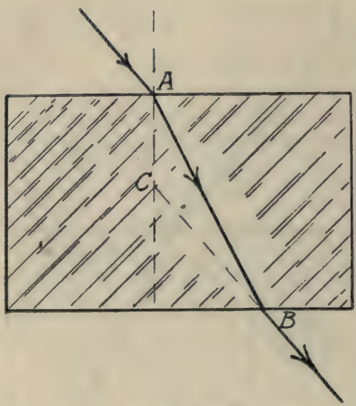

Fig. 65.

If the experiment had been conducted under conditions which would have allowed the ray of light to pass from a vacuum to the glass, the result would have been very slightly different. The difference, however, is so small as to be wholly negligible for most practical purposes.

Again, the refractive index of a substance depends to some extent on the colour of the light used, being least for red and greatest for violet light. A very accurate determination on a specimen of crown glass gave a value of 1·5137 for the refractive index when red light was used, and a value of 1·5331 when violet light was used. The difference it will be seen, is only about $1\frac{1}{4}$ per cent. of the whole.

Tables of refractive indices usually give the value of yellow light, which is about the mean of the extreme values.

Fig. 65 shows a method of finding the refractive index which may be employed instead of the construction given in Fig. 64. The path of the ray is traced in and out of the glass block. The line representing the ray at exit is then produced backwards until it cuts the normal as shown in the figure. The ratio $\frac{AB}{CB}$ is the refractive index. Students with a little knowledge of geometry will find it an interesting problem to prove that the constructions shown in Figs. 64 and 65 give the same result.

Referring again to Fig. 63, the student should observe that if the block of glass is rectangular, or if the opposite faces are parallel, the ray of light FK is parallel to AB. This should be verified by an examination of the various figures obtained in Experiment 34.

Passage of a Ray of Light through a Prism.—In Fig. 66, ABC indicates a prism of glass whose section is an equilateral triangle. If a ray of light fall on one face of this prism in a direction DE, it will, on entering the glass, be refracted *towards* the normal and follow a path EG.

At G it passes from glass to air, and it will consequently be refracted *away* from the normal, as CJ.

Now it has already been mentioned that the refractive index of a medium depends to some extent on the colour of the light used. Hence, if $DECJ$ represent the path of a ray of red light, it is clear that if violet light were employed the angle through which it would be refracted would in both cases be greater and its path would have been similar to $DEKL$.

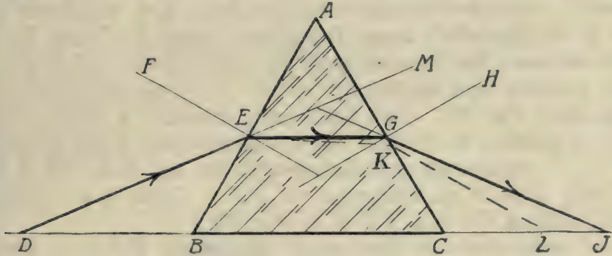

FIG. 66.

Now if a beam of white light is passed through a prism in the direction of DE in Fig. 66, a band of colour is produced, commencing with red at J and passing through the other "colours of the rainbow"—orange, yellow, green, blue, indigo—and apparently ending with violet at L.

This indicates that "white" light (for example, sunlight) is composed of many colours, and if light of these colours be suitably merged together it is possible to produce white light. This band of colour is called the continuous spectrum.

Experiment 35.

Trace a ray of light through a prism of glass by means of pins, placing pins at DEG and J, so as to appear in a

straight line when the eye is placed beyond J. The student will note that the pins at D and E appear fringed with colour. This is due to the fact that the pins are reflecting more or less white light.

By producing the line DE and producing JG to meet it at N, we are able to measure the angle MNJ, which is the angle through which the ray has been deviated by its passage through the prism.

Repeat the experiment several times, using different angles of incidence. Make a table showing the relation between the angle of incidence and the angle of deviation. What are the conditions which give minimum deviation?

Total Internal Reflection.—When light passes from glass or water or other similar media into air, we have seen that it is deviated *away* from the normal. That is, the angle between the normal and the path of the ray is greater in air than it is in water.

Consider Fig. 67. Let light pass through a point P in a block of glass. If it meet the surface of the glass normally, as PA, it passes on without deviation, as AE. If it take the path PB, it is refracted to BF. (This is a repetition of the case of the ray BFK in Fig. 63.)

Now it is obvious that since the angle FBK is greater than the angle PBJ there will come a time, as the ray within the glass becomes more oblique, when the angle between the external ray and the normal is a right angle.

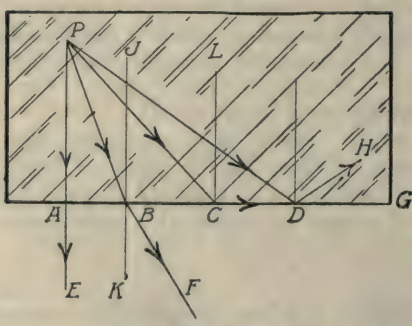

Fig. 67.

This condition is reached by the ray PC, which is refracted so that it passes along the surface of the glass in the direction CG. The angle PCL is called the "critical angle."

If this angle is ex-

ceeded, as in the case of PD, the ray fails to pass into the air and "total internal reflection" occurs, as DH.

This mode of reflection of light is the most perfect known and is utilised in the reflecting prism which is shown in Fig. 68. It consists of a glass prism whose section is a right-angled triangle.

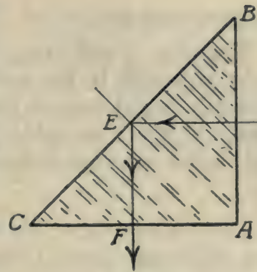

FIG. 68.

If light strikes the side of the prism, represented by AB, normally, it passes into the glass without deviation, as shown by the ray DE. This light has an angle of incidence on the hypotenuse BC which is above the critical angle. Hence, the whole of the light is reflected in the direction EF, and since it strikes the side CA of the prism normally no deviation is produced by the exit of the light from the glass.

The following table showing the indices of refraction of a few common media will be useful for reference:

Substance.	Refractive Index.
Glasses—	
Boro-silicate crown . . .	1·510
Soft crown	1·515
Hard crown	1·518
Barium crown	1·607
Light flint	1·547
Dense flint	1·963
Quartz	1·544
Carbon bisulphide	1·629
Water	1·333
Ice	1·309

Experiment 36.

Examine a reflecting prism. Trace the path of a ray of light through it by means of pins. What happens if the incident ray does not fall on the first face of the prism normally ? Is it possible to select an angle of incidence so that total internal reflection does *not* occur ?

Exercises 11.

1. Give an account of any experiment which you have carried out which proves the following general law for refraction :—When light travels obliquely from one medium into another in which the speed is less, it is bent towards the perpendicular, and when it passes from one medium to another in which the speed is greater, it is bent away from the perpendicular produced into the second medium.

2. A penny was placed at the bottom of a bowl in such a position that the edge just hid the coin from the observer. Explain why it becomes visible when water is poured into the bowl.

3. Why does a stick held in water appear bent to the observer ?

4. A ring is at the bottom of a pond one foot deep and lies two feet from the observer. Give a diagram to scale, showing its apparent position to the observer.

5. A boy holds a penholder at the back of a piece of glass and looks through the glass obliquely. Give a diagram to show what he sees.

6. A diver is being lowered into the sea and the life-line is held vertically above him. Give a drawing to show how the line looks to the diver.

7. In a house there are two windows, one being made from glass having a plane surface and the other being made from cheap glass, having a surface which is not quite plane, but has small depressions in it. Explain why a person in the street looks distorted through one window and not through the other.

8. Explain why the reflections of objects seen in cheap looking-glass give distorted images.

9. The index of refraction may be defined as the ratio of the speed of light in air to its speed in any other medium. The following refractive indices are given :—Water 1·33, alcohol 1·36, benzene 1·50, and flint glass 1·67. Calculate the speed of light in these media.

10. Rays of light proceed from air into water, making angles of 10°, advancing by 10° to 40°. Give drawings and mark clearly the normal angle of incidence and angle of refraction in each case. Take the index of refraction of water as 1·33.

11. Draw any horizontal line XY and on this line construct a circle of 3″ radius. Through the centre R draw a vertical diameter AA', and in the top left-hand quadrant draw radii RO, RB, RC, making angles of 10°, 20° and 30° with the normal AA'. Drop perpendiculars from O, B, and C on to AA'. Make three triangles in the bottom right-hand quadrant in which the perpendiculars dropped from O', B', and C' are equal to the corresponding perpendiculars divided by 1·5.

If the refractive index for benzene is 1·5, which angles represent the angles of incidence and refraction in this construction ?

12. Opaque objects, such as metals and alloys, when examined by the metallurgical microscope must be examined by reflected light. Vertical or oblique illumination may be used. To produce vertical illumination the following methods have been tried :—

(*a*) A small annular silver mirror forming an angle of 45° with the axis of the microscope.

(*b*) A semicircular mirror mounted as above and partially covering the objective.

(*c*) A very small central mirror mounted as in (*a*).

d) A totally reflecting right angled prism, covering half the aperture of the objective.

(*e*) A plain glass disc,

Assuming a highly polished specimen and a horizontal beam of light, show diagrammatically in each case how the light is reflected to the eye. The beam of light is admitted to the tube of the microscope through a hole in its side, is reflected downwards by the reflector, through the objective, and thence is reflected back to the eye by the object.

13. Air changes in density as the pressure and temperature vary. When high atmospheric altitudes are reached the index of refraction of the air becomes less, owing to the diminution in density. Show by diagrams the real and apparent positions of a star, (*a*) at its zenith, (*b*) just below the horizon.

14. Explain, by means of a diagram, the term "critical angle."

15. A coin lies at the bottom of a pond which is 2 feet deep. Explain by means of a diagram why it appears at a less depth to an observer in a boat. Calculate its apparent depth from the formula given :—

$$\text{Index of refraction} = \frac{\text{Real depth of the water.}}{\text{Apparent depth of the water.}}$$

16. If a body be viewed through a plate of transparent material, it appears to be nearer to the observer than it really is. Taking the following formula :—

$$\text{Index of refraction } (\mu) = \frac{\text{Real thickness of medium}}{\text{Apparent thickness of the medium}}$$

calculate the apparent depth or thickness of the following media, assume that the real depth in each case is 6″ :—Indices of refraction, diamond 2·44, flint glass 1·58, crown glass 1·5, carbon bisulphide 1·68, water 1·33.

17. Taking other media, give a simple illustration to show that the following statement is likely to be true :—"Rays of starlight passing through the atmosphere are refracted or curved downwards. The effect is to make objects appear higher than they really are."

18. Give a description of the simplest form of heliograph with which you are familiar.

19. During an ordnance survey it was necessary to pick out a distant station. A cone of burnished tin was hoisted on a pole. Explain how this would help.

20. A beam of light is directed through rectangular vessels with glass sides containing the following liquids :—water, alcohol, and cedar wood oil. Show diagrammatically the path of the beam given the following refractive indices :—water 1·333, alcohol 1·36, and cedar-wood oil, the same as glass.

CHAPTER XII

IMAGES

The Formation of Images.—Everyone is familiar with the formation of an "image" on a screen, for we see it in the projection of an optical lantern and its modern development the cinematograph. A magnified "image" of an object is also seen when one applies the eye to a telescope or microscope. We will now consider the formation of images, first by the reflection, and second by the refraction of the rays of light which emanate from the object.

The Focal Length of a Spherical Mirror.—Consider a concave spherical mirror indicated by the arc ABC in Fig. 69.

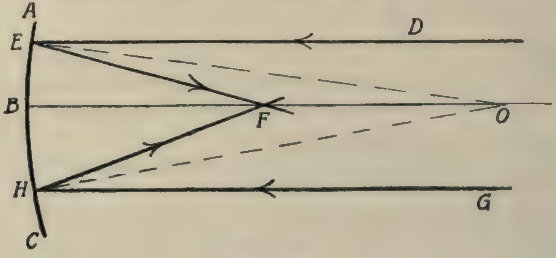

FIG. 69.

O is the centre of the sphere. The line passing through O and the middle (B) of the mirror is called the *axis* of the mirror.

If an incident ray DE is parallel to the axis it will be reflected along a path EF such that the angles of incidence and reflection are equal. EO is, of course, the normal.

Provided that the arc ABC is only a very small portion of the whole sphere (in other words, if the curvature of the mirror is not very great) it is found that all incident rays which are parallel to the axis are reflected along lines which pass through the common point F. This point is called the *focus*, and the distance from the mirror to the focus (namely, BF) is called the "focal length" of the mirror.

In Fig. 70 ABC represents a convex spherical mirror, BO is the axis and O the centre of the sphere. The incident ray DE is reflected along EJ (OE produced being the normal).

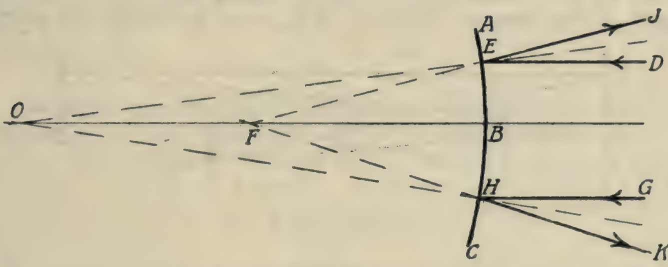

FIG. 70.

Now it is obvious that, since these rays are divergent, they will never meet in the direction in which they are travelling. But if the lines are produced backwards, they meet at F. So F is the focus of this mirror and BF is its focal length.

It is necessary to emphasize that this convergence of parallel rays to a point only occurs when the mirror is a very small portion of a sphere. If this condition is fulfilled, the focal length is found to be half the radius of curvature. Hence in Figs. 69 and 70, BF is half the length of BO.

Real Images.—In Fig. 71 a concave mirror is shown having its focus at F, and the centre of the sphere at O. A bright object is placed in front of the mirror, the object being indicated in the diagram by the arrow AB.

We may assume that rays of light are being sent out from this object in all directions. Some of these rays will fall on the mirror. Consider the ray AC. This, being parallel to the axis of the mirror, will be reflected in a direction CD, passing through the focus F. Another ray passing from A will strike the mirror at the middle point E, and will be reflected along EG (the angle of incidence AEO being equal to the angle of reflection GEO). These lines intersect at H, and if other rays from A were drawn, careful construction would show that they are all reflected along lines passing through H.

H, we may therefore assume, is the position of the arrow-head in the image. The ray BE, since it strikes the mirror

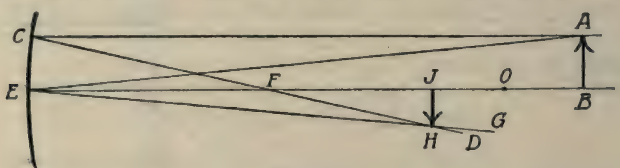

Fig. 71.

normally, will be reflected back along its own course. The position of the tail of the arrow is therefore obviously at J.

Experiment 37.

A 4-volt electric bulb forms a very convenient "object" for experimental work in optics. Failing this a light may be enclosed in a box and the rays allowed to escape only through a hole about half an inch square, the hole being covered with wire gauze.

Various forms of mirror-holders are purchasable, but it should not take long for an ingenious student to see the possibilities of a cork and a piece of plastic wax or clay. A suitable screen consists of a piece of white card about 3 inches square. It may be mounted in a split cork.

Now place the "object" on a table in a darkened room and place a concave mirror about a foot in front of it. Move the screen to and fro until the "image" is seen. If an

electric bulb is being used the image should be the filament of the lamp. By a careful adjustment of the screen the image may be "focussed," that is, it may be made to appear quite sharp and well-defined.

It is obvious that the screen must not obstruct the rays of light passing from the object to the mirror, hence it is necessary slightly to twist the mirror so that its axis passes between the object and the screen.

Measure the distance from object to mirror, and from mirror to image. Also measure the length of the object and image. Repeat the experiment several times, placing the object at a different distance from the mirror in each case. Tabulate the measurements mentioned above for future reference.

The Spherometer.—This instrument is employed for the determination of the radius of curvature of spherical surfaces. It consists of a very small steel tripod whose feet form an equilateral triangle. A fourth foot which forms the base of a micrometer screw passes through the middle of this triangle.

The instrument is first placed on a sheet of plate glass, and the micrometer adjusted so that the central (movable) foot just touches the glass. It is then transferred to the spherical surface and the micrometer again adjusted for contact. The difference in the micrometer readings gives the difference between the level of the tripod feet and that of the micrometer foot, due to the curvature of the surface.

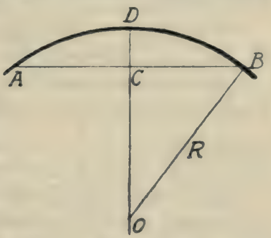

FIG. 72.

In Fig. 72, let ADB represent the curved surface. Let B represent the point of contact of one of the tripod feet, and let D be the point of contact of the micrometer foot.

CD = difference of micrometer readings. Denote this by h.

CB = distance from a tripod foot to the micrometer foot, which may be measured on the spherometer. Denote this by d.

OB=radius of the sphere. Denote this by R. Now OD is another radius, and hence $OC=(R-h)$.

In the right-angled triangle OCB we have :—

$$R^2=(R-h)^2+d^2$$

$$i.e.\ R^2=R^2-2Rh+h^2+d^2$$

$$\text{Hence } 2Rh=h^2+d^2$$

$$\text{and } R=\frac{h^2+d^2}{2h}.$$

It will be recalled that we have confined our consideration of spherical mirrors to those having very little curvature. Hence in applying a spherometer to them, h will be a very small quantity, and in most cases the term h^2 may be omitted without introducing any appreciable error. The formula in that case becomes: $R=\dfrac{d^2}{2h}$

Experiment 38.

Measure the radius of curvature of the mirror used in Experiment 37. Use the formula containing the term h^2, and then find the percentage error introduced by omitting this term.

Returning to the table of results obtained in Experiment 37, let the distance from the object to the mirror be called u, and the distance from the mirror to the image be called v. Then denoting the focal length of the mirror by f it can be shown that: $\dfrac{1}{f}=\dfrac{1}{u}+\dfrac{1}{v}$

The focal length of the mirror should be calculated for each case, and the mean of these results obtained. We have already seen that the focal length of a spherical mirror is half its radius of curvature, so that the focal length obtained by the optical method may be compared with that given by the spherometer measurement.

When an image can be thrown on a screen, it is said to be a "real" image. In Fig. 71, the image may be described as: "real, diminished and inverted."

Virtual Images.—We will now consider the behaviour of a *convex* mirror. This is shown in Fig. 73. Again AB represents the object, and F the focus. The ray AC being parallel to the axis is reflected in the direction CD and the ray AE passes along EG (the angle AEB being equal to the angle BEG).

Now these two reflected rays are divergent, and will therefore meet only if produced backwards. The intersection takes place at H, which determines the position of the head of the image.

Since this image is behind the mirror, it is obviously impossible to throw it on to a screen, and hence it cannot be called "real" in the sense that the last image was.

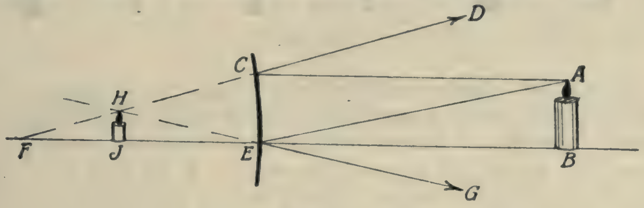

Fig. 73.

This is a case of a "virtual" image. If the eye is placed in front of the mirror, the virtual image is seen, and in the case illustrated in Fig. 73 the image may be described as "virtual, diminished and erect."

Lenses.—A lens is a piece of transparent material (usually glass) having a curved surface (usually spherical). Fig. 74 shows the form in section of a number of common lenses. In A both surfaces are convex, and such a lens is called a "bi-convex lens." In B both are concave, and hence it is called a "bi-concave lens." In C and D one surface is flat (or plane) and the other spherical. C is a "plano-convex lens," and D a "plano-concave lens." In E and F both surfaces are spherical, but one is concave and the other convex. Such lenses are known as "concavo-convex lenses."

In our consideration of lenses we shall assume that in all cases the curvature is very small, and the lens itself very thin.

Of course there are lenses which do not fulfil these conditions, but their behaviour is somewhat complex, and the study of such lenses is best delayed until the student has had experience with those of the simpler type.

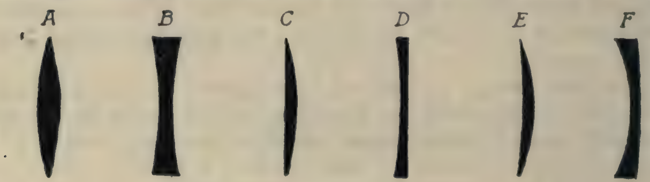

Fig. 74.

Lenses are conveniently divided into two classes: (1) Those which cause a parallel beam of light to converge are called convergent lenses. These are thicker at the axis than they are at the edge of the lens. *A*, *C*, and *E*, in Fig. 74, are convergent lenses. Those which are thickest at the edge, such as *B*, *D*, and *F*, in Fig. 74, are called divergent lenses, because they cause a parallel beam of light to diverge.

The Focal Length of a Lens.—In Fig. 75 *AB* represents a bi-convex lens, which is, of course, convergent. A line drawn through the centres of the spheres of which the curved surfaces of the lens are portions is called the axis of the lens. *CD* and *EG* represent rays of light whose paths are parallel to the axis of the lens.

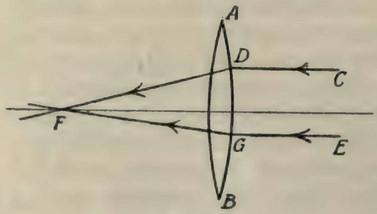

Fig. 75.

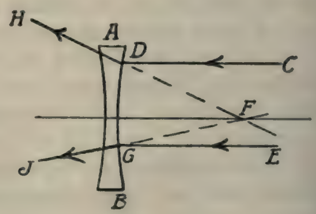

Fig. 76.

When these rays pass into the lens they are refracted in a manner similar to that in which a ray is refracted by a prism. It is found that all such rays pass through a point F on the axis, called the focus of the lens. The distance from the lens to the focus is called the focal length of the lens.

In Fig. 76, AB represents a divergent lens. In this case the rays CD and EG, which are parallel to the axis, are refracted by the lens so that their paths diverge as shown. All such rays, however, on being produced backwards pass through the common point F, which is the focus of this lens, and again the distance from the lens to the focus is called the focal length.

Strictly speaking, part of the deviation of a ray of light passing through a lens takes place at the front surface and part at the back surface of the lens. This is not shown in the diagrams.

Images Formed by Lenses.—In Fig. 77 let AB represent a convergent lens of which the focus lies at F. Let CD be an object. Consider a ray passing from C parallel to the axis. This will be refracted through the focus. Another ray from C is shown passing through the middle of the lens.

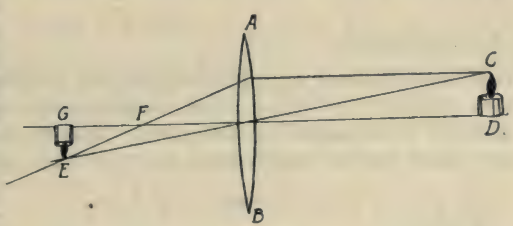

Fig. 77.

If, as we have assumed, the lens is thin, and not deeply curved, this ray suffers practically no deviation, but passes on in a straight line as shown.

These two rays meet at E, and this is the position of the head of the image.

Experiment 39.

Mount a convergent lens, a screen and an "object" in a darkened room as in Experiment 37. The screen and

object must, of course, be on opposite sides of the lens. Again measure the distance u (from object to lens) and v (from lens to image) for a number of cases. If f is the focal length of the lens calculate its value in each case and obtain the mean. The formula: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ is still true as in the case of the concave spherical mirror.

Experiment 40.

Having obtained the focal length of a convergent lens in Experiment 39, take the same lens into the sunlight (which may be regarded as parallel rays of light). These rays converge to a point, which appears as a bright spot of light. This is, of course, the focus of the lens, and the focal length may, therefore, be measured directly.

Since the lens concentrates the sun's *heat* rays as well as the light rays, the spot of light is likely to be *hot*. The student should have no difficulty in testing whether this is so.

Experiment 41.

Using the same lens, hold it near the wall of a room opposite the window. With a little adjustment an image of the window is obtained on the wall. This is very similar to the darkened room experiment except that the distance u is very great, and consequently $\frac{1}{u}$ is very small indeed and may be neglected without serious error.

Hence the formula becomes: $\frac{1}{f} = \frac{1}{v}$ and $f = v$.

The distance from the lens to the wall is, therefore, the focal length of the lens.

The student is now in a position to verify the fact that the ratio of the sizes of object and image is the same as the ratio of their respective distances from the lens or mirror. In other words the "magnification" of the image is equal to $\frac{v}{u}$.

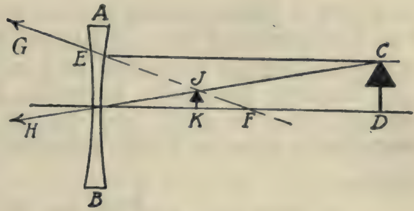

FIG. 78.

If this fraction is less than unity the image is diminished. A magnified image will be indicated by this ratio being greater than unity.

A divergent lens is indicated by AB in Fig. 78, the focus of which is shown at F. CD is an object as before. The ray from C which is parallel to the axis is refracted in the direction EG (note that GE produced passes through F). The ray from C which passes through the middle of the lens proceeds in the direction of H without deviation.

Intersection takes place at J, and JK is therefore the image. Note that it is virtual (and therefore cannot be thrown on a screen, but if the eye be placed between G and H it can be *seen*), diminished and erect.

The Magnifying Glass.—Referring back to Fig. 77 it will be seen that the distance from the object to the lens was greater than the focal length of the lens. We will now find the effect of making this distance less than the focal length.

Fig. 79 shows the lens AB with its focus at F. CD is the object. Using the same construction we find that the

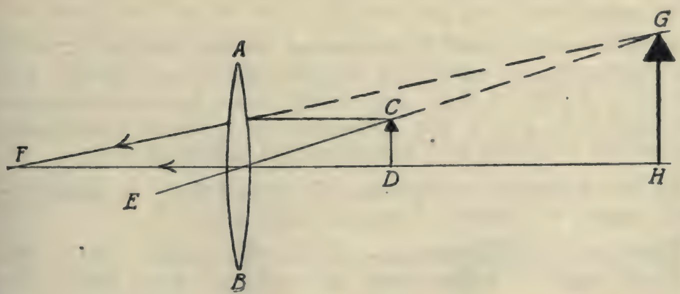

FIG. 79.

rays which before converged to form a "real" image are now divergent and the virtual image GH is produced. Note that this image is magnified and erect.

If the eye be placed between F and E this magnified virtual image is seen, and this is the explanation of the magnifying glass or simple microscope.

Exercises 12.

1. Draw a concave mirror of 4″ radius, and with a 2″ arc. Mark on the drawing (*a*), (*b*), and (*c*) as defined.

(*a*) Centre of curvature is the centre of the sphere of which the mirror forms part.

Bisect the arc of the mirror and join to the centre. The point where the radius cuts the arc or middle point of the mirror is called the *pole* (*b*).

(*c*) The *principal focus* is midway between the centre of curvature and the *pole*. The *focal length* is the distance from the pole to the *principal focus*, and if the focal length of the mirror is called f and the radius of curvature r, insert measurements in your drawing and show that $r=2f$.

2. Repeat the above example for concave mirrors of 3″, $3\frac{1}{2}$″, and 5″ radius with 2″ arcs.

3. Take concave mirrors of 2·75″, 3·25″ and 3·375″ radius and in each case with arcs 2·5″ long show where rays parallel to the principal axis intersect the principal axis after reflection.

4. A luminous point O is situated on the principal axis of the mirrors in the preceding question and lies 1″ behind the centre. A ray of light proceeding from this point strikes the mirror 1″ above the principal axis. Show graphically where the reflected ray intersects the principal axis.

5. The following information is supplied in a catalogue with regard to a series of concave mirrors :—

Diameter in millimetres .	50	63	75	100	150
Focal length in millimetres	75	75	80	100	180

Determine graphically in each case the radius of curvature.

6. Similar information to that of the preceding example is supplied in respect of convex mirrors :—

Diameter in millimetres . .	50	63	75	100
Focal length in millimetres . .	75	75	80	100

Find the radius of curvature in each case.

7. Measurements were taken of a number of spherical watch-glasses and the following readings were noted :—

	Spherometer readings in mm.	Chordal diameter in cm.
Case 1 . .	4·26 —4·42	6·4
Case 2 . .	6·88	5
Case 3 . .	1·49 —1·42	10
Case 4 . .	1·99 —2·19	11

Taking the distance CB as 2·25 cms. calculate the inside and outside radius of the sphere which forms the watch-glass. Make a scale drawing in each case.

8. In case 1, 3 and 4 of the above question, the inside of the watch-glass was blackened. The outside curve was then used as a convex mirror. Show graphically how a ray of light parallel to the principal axis is reflected in each case.

9. Make sketches of any spherometer you have used and name the different parts.

10. The following results were obtained with a spherometer : Distance from tripod foot to micrometer foot, 3·1 cms. Difference of micrometer readings for a concave mirror, 0·15 cms. ; difference of micrometer readings for a concave mirror, 0·127 cms. Calculate the radius of curvature for each mirror.

11. Make sketches of reflectors used for bicycle, motor car and railway lamps and electric light. Give sectional views wherever possible and state what are the approximate shapes.

12. The inside of each of the watch-glasses in Question 7 is blackened and they are then used as convex mirrors. Show how a ray of light 1 cm. from the principal axis is reflected in each case. Neglect the slight difference of curvature between the two surfaces.

13. A lens which changes a parallel beam of light into a convergent beam is called a "converging or convex lens." These lenses are always thickest in the centre. Draw and name three types.

14. A lens which changes a parallel beam of light into a divergent beam is called a diverging or concave lens. These lenses are always thinnest in the centre. Draw and name three types.

15. Define the "focus" of a lens.

16. An experiment was performed to find the focal length of a convex lens. The lens was fixed and a pin was placed at some distance from it Another pin was placed on the other side of the lens until the image and the pin-head appeared together. The following results were obtained :—

u, distance from object 49·2 cms. v, distance from image 22·5 cms.
" " " 35·6 " " " " 25 cms.

Determine the focal length of the lens.

17. This experiment was then performed with the object nearer to the convex lens than the principal focus. Give drawings, showing the image of the pin in this case. Is the image real or virtual ?

18. An experiment was performed to determine the focal length of a convex lens. The lens was placed in front of the screen and a strongly illuminated pin was placed at the other side of the lens The lens was moved until a sharp image was obtained on the screen. The following results were obtained :—

u, 42 cms., u, 61·5 cms. v, 67 cms., v, 43·3 cms.

Determine the focal length of this lens.

19. Describe, with the aid of sketches, why the image of a pin as seen in a plane glass mirror differs from that seen on the ground-glass screen of a camera.

20. Why is a magnifying glass sometimes called a "burning glass"? Show how you would find the principal focus of a lens by means of the sun.

21. An arrow $1\frac{1}{4}''$ long is $2''$ from the pinhole in a pinhole camera. If the arrow is vertical, show by means of a diagram the image formed on a screen $3''$ away from the pinhole.

SECTION V.—HEAT

CHAPTER XIII

THERMOMETRY

Heat and Cold.—From our early days we are accustomed to associate certain sensations with heat and cold. Such sensations are, however, not very trustworthy, for in certain circumstances two men may not be in agreement about the heat (or coldness) of a body.

Experiment 42.

Take three basins, and into No. 1 put hot water. In No. 2 put some ice-cold water and fill No. 3 with lukewarm water.

Now place the left hand in the hot water in No. 1, and the right hand in the cold water in No. 2, and leave them there for one or two minutes.

Then put both hands into the tepid water in No. 3. This water will feel cold to the left hand and warm to the right hand.

This experiment shows how unreliable is the sense of touch as an indication of heat or cold.

Because the left hand was hotter than the warm water, heat flowed from the hand to the water and the hand felt the loss of heat as a sensation of coldness. The right hand being colder than the water, heat flowed into it from the water, and this acquisition of heat gave the sensation of warmth.

Temperature.—The degree (or intensity) of heat (which is quite distinct from the *amount* of heat) is commonly denoted by the term " temperature."

Experiment 43.

Obtain a cylindrical rod of metal with trued ends, also a piece of sheet metal cut to the form shown in Fig. 80, the dimension of which are such that the rod will just pass into the gap in the sheet metal and the end of the rod will just enter the circular hole.

Now hold the rod in the flame of a Bunsen burner for a minute or two. It will be found that it will neither pass within the gap nor enter the hole while it is hotter than the

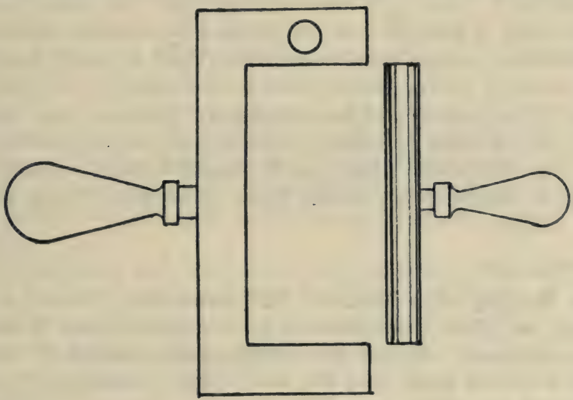

Fig. 80.

sheet metal, but will do so when it has cooled to approximately the same temperature again.

The two foregoing experiments establish two facts :—
(1) When two bodies, of different temperatures, are in contact, heat flows from that at the higher temperature to that at the lower temperature, with a tendency to establish equality.

(2) The dimensions of a body depend upon the temperature. In most cases the dimensions increase as the temperature rises.

It is obviously quite possible (though perhaps not very *practicable*) to use a rod of iron as a thermometer. Since

the length of the rod is a function of its temperature, all that is necessary is to measure the length of the rod and determine the temperature corresponding to this length.

This method has two very grave disadvantages :—(i) the iron rod in absorbing heat necessarily reduces the temperature of the body whose temperature it is required to determine ; (ii) the increase in length for any reasonable rise of temperature is so small that accurate measurement is extremely difficult. Moreover, the measure will be affected by heat.

Although a rod of iron is practically useless as a thermometer the general principle described is that on which most modern thermometers are constructed.

For a thermometer to be suitable for common use, it must, firstly, be of such a nature that it absorbs very little heat from the body to which it is applied, and, secondly, its change in length (or volume) must be indicated in a manner easily visible.

Experiment 44.

The Making of a Mercury Thermometer.—Obtain a piece of capillary glass tube about a foot long, and test it for uniformity of bore. To do this suck a short thread of mercury about an inch long into the bore and measure its length as accurately as you are able.

Now gently blow down the tube and force the thread of mercury to a new position and measure it again. Repeat this several times so as to bring the whole length of the tube under examination.

Obviously if the bore is uniform the length of the mercury thread will be constant. If in any part the bore of the tube is increased it will be indicated by a shortening of the length of the thread of mercury, and, on the other hand, if the bore is constricted in any part it will cause the mercury thread to become longer.

If the piece of tube *has* a uniform bore it is fit for use in the construction of a thermometer. If the bore is irregular, the tube is useless.

Experiment 45.

Having obtained a piece of capillary tube which has a satisfactorily uniform bore, hold one end of it in a blow-pipe flame until the end is sealed up. By cautiously blowing down the tube through the other end it is possible while the glass is still hot enough to be plastic to blow a bulb on the sealed end.

The size of the bulb is decided by the type of thermometer desired. A large bulb will give a very large range of temperature record, but a very poor degree of sensitiveness. High sensitiveness and small range are associated with a small bulb.

When the tube is cool enough to handle, the blowpipe flame should be applied at a point about one inch from the open end, and the tube nearly pulled apart. This will produce a constricted part (as shown in Fig. 81) which can easily be sealed up when required.

FIG. 81.

The next step is to introduce some mercury. Most people have experienced the difficulty of filling a narrow-necked bottle under a common water-tap. Water cannot enter the bottle without air passing out, and the narrow neck does not provide a passage for both.

This difficulty is met with in a very aggravated form in the case of the thermometer tube. It is overcome by expelling some of the air first.

By gently heating the bulb the contained air is made to expand and some of it passes out of the tube. If the open end of the tube is now placed in a bowl of mercury, as the heated air within the tube contracts the reduced pressure will cause a little mercury to flow into the tube. With perseverance a few drops of mercury can be forced into the bulb.

When this is accomplished the bulb is held in the flame until the mercury boils. The mercury vapour produced replaces the air originally in the tube, and if the open end of the tube is now held below the surface of some mercury, the

tube will be completely filled with mercury when the enclosed vapour condenses.

The thermometer is now raised to a temperature somewhat in excess of the highest temperature it is desired to record, and a small blow-pipe flame is applied to the constricted part of the tube and the upper portion pulled off. The bulb and tube are now completely filled with mercury and hermetically sealed.

The Principle of the Mercury Thermometer.—As the mercury cools it of course contracts, and thus leaves the upper part of the bore of the stem unoccupied. A very little thought should make it clear that, since the temperature of the mercury influences its volume, the temperature is indicated by the position of the mercury in the stem.

This type of instrument has several merits which will repay a little consideration. In the first place a very small quantity of mercury is employed, and thus it acquires the temperature of the surrounding medium without the absorption of much heat. Hence in most cases there is no appreciable fall of temperature of the body on the introduction of the thermometer.

Secondly, the major part of the mercury is contained in the bulb of the instrument, yet any *increase* in volume is accommodated in the fine bore of the stem. A small increase in volume is therefore indicated by a comparatively large movement of the mercury.

Again, mercury remains liquid over a very large range of temperatures, and thus mercury thermometers are available for a great variety of purposes.

Lastly, experiment has shown that mercury expands very uniformly with a rising temperature. That is, equal rises of temperature are indicated by equal increases of volume.

The Graduation of a Thermometer.—The stems of mercury thermometers are marked off into equal divisions called "degrees," and the numbers by which they are designated have been determined by common practice. There are at the present time two scales with which the student should be familiar.

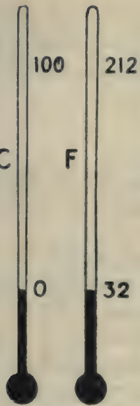

Fig. 82.

There are certain temperatures, such as the melting points of common solids, and the boiling points of well-known liquids which are, so to speak, outstanding features in any temperature scale. Of these the temperature at which ice melts and that at which water boils under normal atmospheric pressure are perhaps the best known.

The Centigrade scale, as its name suggests, has a hundred divisions between the melting point of ice (or, what is the same thing, the freezing point of water), and the boiling point of water. The former is denoted by zero and is written 0° C., and the latter is therefore 100° C.

On the Fahrenheit scale these two temperatures are respectively 32° and 212°. Thus 0° C. and 32° F. are equal temperatures and denote the temperature at which ice and water can exist together.

Experiment 46.

Support a large funnel in a stand, and having filled it with small lumps of ice and placed a vessel to catch the water as the ice melts, place the ungraduated thermometer made in Experiment 45 with its bulb well within the ice as shown in Fig. 83.

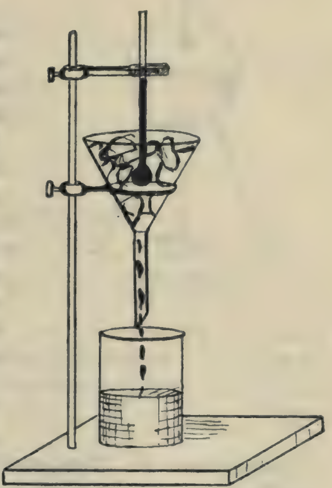
Fig. 83.

Since the experiment is likely to be conducted in a room whose temperature is sufficient to cause the ice to melt, the thermometer will gradually acquire the temperature of melting ice.

Watch the mercury carefully and do not mark its position until no further movement can be detected.

This point on the stem of the

thermometer represents the freezing point of water, and may be marked 0° if a Centigrade thermometer is required, or 32° if it is desired to have it record temperatures on the Fahrenheit scale.

Experiment 47.

Fit a flask with a cork through which two holes have been bored, one to carry the thermometer, and the other a bent glass tube as shown in Fig. 84.

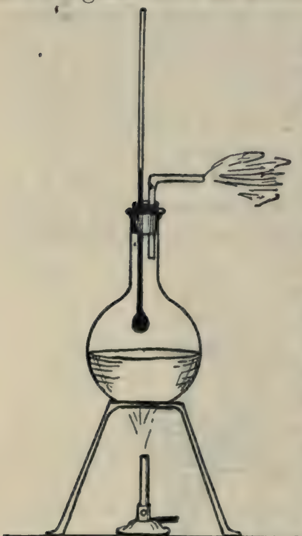

Fig. 84.

Erect the whole on a stand, and having placed a little water in the flask apply the flame of a Bunsen burner until the water boils.

The thermometer and the water should be in such relative positions that the bulb of the former is at least an inch above the surface of the water.

As the steam is formed some of it will condense on the bulb of the thermometer and gradually raise the temperature of the mercury up to that of the condensing point of steam (or boiling point of water).

Wait until the mercury ceases to rise any further, and then mark its position on the stem and designate it 100° C. or 212° F. as the case may be.

The distance on the stem between the freezing point and boiling point of water may now be divided into equal divisions or degrees. Note that there are 100 divisions in this interval on the Centigrade scale and 180 divisions on the Fahrenheit scale.

Conversion of Temperatures.—Generally speaking, the Centigrade scale is used for scientific purposes throughout the world. In this country the Fahrenheit scale is exten-

sively used for commercial purposes, and very largely in engineering work.

It frequently happens, therefore, that a temperature on one scale is required on the other, and thus a ready means of conversion is needed.

It is, of course, only a case of simple proportion, but the student should remember that it is absolutely necessary to work with the number of degrees above or below some fixed point, usually the freezing point of water.

It will be seen that between the freezing point and boiling point on a Fahrenheit thermometer there are 180 degrees, and since these are equal to 100 degrees on the Centigrade scale, it follows that each Fahrenheit degree is equal to $\frac{5}{9}$ths of a Centigrade degree.

Ex. 17. Convert 15° C. and 87° F. to corresponding readings on the other scale.

(1) 15° C. = 15 degrees above the freezing point.

$$\text{and } 1 \text{ degree C.} = \frac{9}{5} \text{ degree F.}$$

$$15 \quad ,, \quad ,, = \frac{9 \times 15}{5} \text{ degrees F.}$$

$$= 27 \quad ,, \quad ,,$$

This 27 degrees F. is 27 degrees *above the freezing point*, which is 32°.

$$\therefore 15^\circ \text{ C.} = 27 + 32 \text{ degrees F.}$$

$$= 59^\circ \text{ F.}$$

(2) 87° F. = 87 — 32 degrees above the freezing point.
= 55 ,, ,, ,, ,,

$$\text{And } 1 \text{ degree F.} = \frac{5}{9} \text{ degree C.}$$

$$55 \quad ,, \quad ,, = \frac{5 \times 55}{9} \text{ degrees C.}$$

$$= 30\tfrac{1}{2} \quad ,, \quad ,,$$

This $30\frac{1}{2}$ degrees is above the freezing point, which is 0° on the Centigrade scale. Thus 87° F. = $30\frac{1}{2}$° C.

A graph may be constructed for these transformations. Since both scales are uniform the graph is a straight line, and it is, therefore, only necessary to fix two points. Fig. 85 shows such a graph in which the two points selected are the freezing and boiling points.

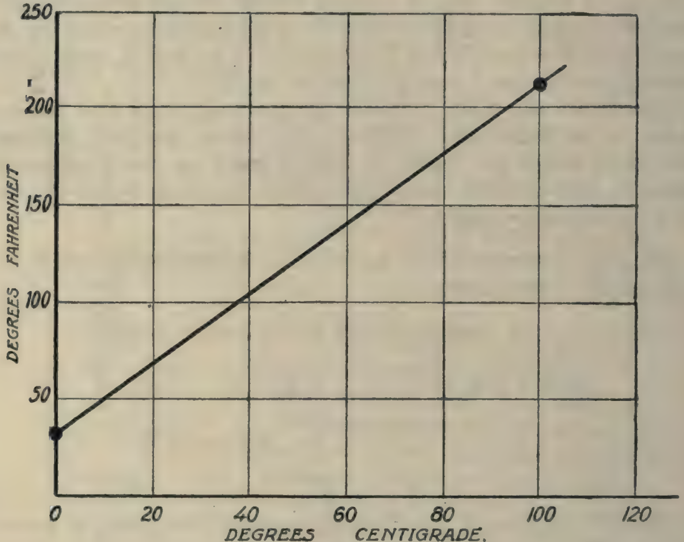

FIG. 85.

The student should reproduce this graph on a large scale, and check the conversions made in Example 17.

The Absolute Scale of Temperature.—We have already seen that the zero point on the Centigrade scale is the freezing point of water. If the thermometer is placed in ice to which a little common salt is added, it will be found that the mercury contracts below this point, showing that a mixture of ice and salt is colder than ice melting in the normal manner.

Suppose such a mixture caused the mercury to fall below the zero point a distance equal to ten degrees on the other

part of the scale. This temperature would be referred to as " minus ten degrees Centigrade " and written $-10°C$.

Ice mixed with some other substances will produce temperatures below that obtained by ice and salt, and certain mechanical means are capable of producing temperatures considerably lower.

Now we shall see later that heat is a form of energy. When we cool a body we take energy from it. It is quite reasonable to suppose that energy cannot be extracted from a body indefinitely. There naturally comes a time when there is none left, and since no chemical or mechanical device can extract what isn't there, the body may be said to be devoid of all heat.

Under these conditions the temperature is said to be " absolute zero." This temperature has not yet been reached, although some experiments have got very near to it. There are many reasons, however, for believing that absolute zero would be reached at $-273°$ C.

For some purposes it is convenient to construct a scale of temperature on which this point is denoted by 0. The size of the degrees is not of so much importance, but the centigrade degree is very suitable.

Such a scale is called the absolute scale of temperature, and it is easy to see that any temperature on this scale is obtained by adding 273 to the reading on the Centigrade scale. Thus 15° C. is equal to 288 degrees absolute. This may be written 288° A.

Engineers sometimes find it desirable to use Fahrenheit degrees on the absolute scale. Since absolute zero on the Fahrenheit scale is $-459°$ F., we obtain the absolute temperature on this scale by adding 459 to the Fahrenheit reading. Thus 32° F. is equal to 491° on this type of absolute scale.

Fig. 86 shows two graphs. One gives the relation between temperatures on the Centigrade and Absolute scales, and the other indicates the relation between Fahrenheit and Absolute temperatures.

These graphs are of interest because they intersect at a point representing $-40°$ C. and $-40°$ F. and 233° A. Thus

—40° C. is equal to —40° F. The student should consider carefully why this is so.

Other Thermometers.—Mercury is not the only substance that can be used in a thermometer. For very low tem-

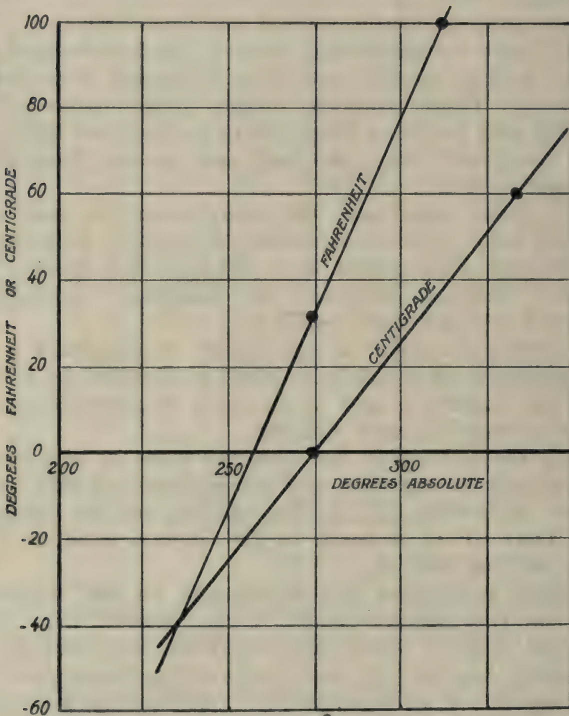

Fig. 86.

peratures mercury is unsuitable, since it freezes at —39° C. Thermometers for low temperatures are often filled with alcohol (usually coloured with a little dye in order that it may be easily seen).

For extremely low temperatures, and for temperatures

above a few hundred degrees Centigrade, thermometers of this type are not suitable. Various electrical and optical devices are employed for the determination of such temperatures.

The following table is of interest and may be useful for reference :—

Absolute zero	$-273°$ C.
Hydrogen boils	$-253°$
Oxygen boils	$-183°$
Carbon dioxide boils . . .	$-78°$
Mercury freezes	$-39°$
Water freezes	$-0°$
Water boils	100°
Tin melts	232°
Lead melts	327°
Mercury boils	357°
Zinc melts	419°
Sulphur boils	445°
Aluminium melts	657°
Zinc boils	918°
Silver melts	961°
Gold melts	1062°
Copper melts	1083°
Pure iron melts	1500°
Silica softens . . . about	1500°
Platinum melts	1750°
Tin boils	2270°
Tungsten melts	3000°

Exercises 13.

1. Draw a line 5″ long and graduate one side to represent the graduations of a Centigrade thermometer ; let the graduations rise from 0° C. to 100° C., rising by 10° C ; the graduations on the other side to represent from 32° F. to 212° F., rising by 20° F.

2. Repeat the above exercise with the graduations on one side from 32° F. to 150° F., rising by 20° F., and corresponding Centigrade graduations on the other side.

3. Draw a line 4″ long and graduate one side to represent from 0° F. to 150° F., graduated every 20° F. On the other side draw a suitable Centigrade scale to correspond.

4. Repeat the above exercise, but graduate one side from 10° C. to 80° C., rising by 10° C. On the other side draw a suitable Fahrenheit scale.

5. Describe, with sketches, the construction of an ordinary mercurial thermometer.

6. Define the terms "freezing point" and "boiling point" of a thermometer.

7. Describe how you would determine the "fixed points" on a mercury thermometer.

8. By means of a large-scale graph of your own construction, convert the following temperatures :—120° C., 135° C., 87° C., 36° C., 25° C. to Fahrenheit readings. Convert the following readings Fahrenheit into Centigrade :—29°, 36°, 65°, 85°, 115°, and 205°. Check each result by calculation.

9. Why are thermometer tubes usually made with a very fine bore?

10. Convert all the temperatures given in the table at the end of this chapter into degrees Fahrenheit.

11. State in your own words what you understand by the word "temperature."

12. A student is performing an experiment, and to take readings of the thermometer he lifts it out of the liquid whose temperature is required. Criticise his method.

13. In 1911 Kamerlingh Onnes liquified helium and the temperature attained was —271·3° C. How far was he from the absolute zero of temperature?

14. The normal temperature of the human body is 98·4° F.; the normal room temperature is 68° F. Convert these temperatures into degrees C.

15. "Segar cones" (fusion thermometers) are used for measuring high temperatures, and the following are some cone numbers and their estimated softening-points :—

Estimated softening-point in degrees C.—	590	620	710	800	920	950	1090	1150	1250
Cone numbers—	022	021	018	015	011	010	03	1	6

Give these temperatures in degrees Fahrenheit and degrees absolute Centigrade.

16. White and Taylor give the following colour scale :—

Name of the colour.	Degrees C.
Dark red heat	566
Dark cherry red	635
Cherry, full red	746
Light red	843
Orange	899
Light orange	941
Yellow	966
White	1,205

Give these values in degrees F.

17. The following boiling points of non-metallic elements are given :—Argon, $-186°$ C. Chlorine, $-33·6°$ C. Krypton, $-151·7°$ C., and Neon, $-239·0°$ C. Express these results in degrees F. absolute.

18. The following figures are taken from a steam table :—

Absolute pressure in lbs. per sq. in.	Temperature F.	Pressure in atmospheres.	Temperature C.
14	209·55		
14·7	212		
30	250·3		
90	320·3		
100	327·8		

Complete the table.

19. The critical temperature of a gas is that temperature above which no pressure suffices to produce a liquid. Convert the following table of critical temperatures in degrees C. and critical pressures in atmospheres into critical temperatures in degrees F. absolute and pressures in lbs. per sq. in.

Substance.	Critical temperature in degrees C.	Critical pressure in atmospheres.
Argon . .	—122·44	48
Oxygen . .	—118·8	50·8
Nitrogen .	—145·1	33·6

20. In rivetting the end of the rivet is made slightly tapered and the hole larger than the rivet. Why is this ?

21. Stromeyer points out that a fresh charge of fuel thrown on to a fire must be brought to the following temperatures before chemical action will take place :—

Dried peat, 435° F. Anthracite dust, 570° F. Lump coal, 600° F., and coke 800° F. Convert these temperatures into degrees C.

CHAPTER XIV

EXPANSION OF MATTER BY HEAT

Linear Expansion.—We have already seen in a general way that bodies mostly expand when their temperature is raised. We will now consider this matter in a little more detail.

In an investigation of this nature, it is always desirable to inquire into the simplest cases first. Hence we will commence with solids, and even so we will consider only their "linear" expansion, that is, their expansion in *length*.

A great many types of apparatus are in use for the measurement of the expansion of metal rods. That shown in Fig. 87 is a suitable form for the purpose.

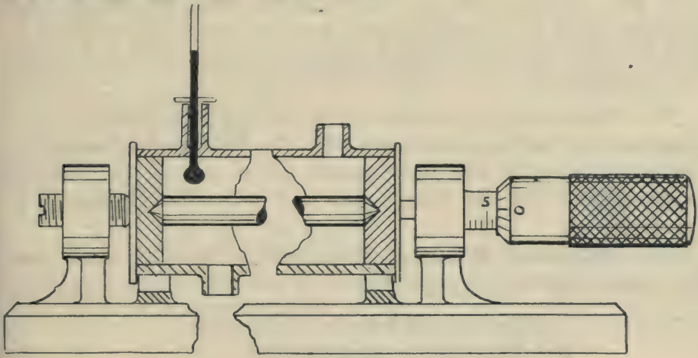

FIG. 87.

The metal rod under investigation is generally about 50 cms. long and has pointed ends. These fit into conical recesses in the metal plugs which are shown in Fig. 87 at the ends of a metal cylinder.

This containing cylinder is lagged with string or other material to prevent undue loss of heat. It is mounted on a stand so that one plug rests against a screw stop which is shown at the left-hand side of the figure.

The other plug is in contact with a micrometer screw gauge, which enables any movement to be determined with accuracy.

The cylinder is fitted with three side tubes. Into the middle one a thermometer is fixed and the other two are used for maintaining a passage of steam through the cylinder.

In use the initial length and temperature of the metal rod are first determined. The rod is then put into the cylinder and the plugs pushed in until the pointed ends of the rod pass into the conical recesses provided for the purposes.

The cylinder having been placed in its support, the micrometer screw is brought into contact with the right-hand plug and the reading taken. The micrometer screw is then brought back a few millimetres to allow room for expansion, and steam is passed through the instrument for ten minutes or thereabouts.

The thermometer should now record a constant temperature not far removed from 100° C. Without interrupting the steam supply the micrometer screw should be brought into contact with the plug again and its reading taken.

The Coefficient of Linear Expansion.—It is obvious that the actual amount of expansion of a rod will depend upon :—

(i) The initial length of the rod.
(ii) The rise in temperature.
(iii) The properties of the material.

Now it is the last of these conditions that we wish to investigate. Hence we generally calculate from the data provided by the experiment the amount of expansion which one may expect from a rod of unit length, when its tem-

perature is raised one degree. This quantity is called the "coefficient of linear expansion."

Ex. 18. Find the coefficient of linear expansion of brass from the following data :—

Initial length of brass rod . .	50 cms.
Initial temperature of rod . .	17° C.
Temperature recorded by thermometer in steam chamber	99° C.
Initial reading of micrometer . .	0·78 mm.
Final reading of micrometer . .	1·55 mm.

We see that the amount of expansion is $1 \cdot 55 - 0 \cdot 78 = 0 \cdot 77$ mm. Converting this to centimetres we get 0·077.

The rise of temperature was $99 - 17 = 82$ degrees. Assuming that the expansion is uniform we have an expansion of $\frac{0 \cdot 077}{82}$ cms. for each degree.

But the rod was 50 cms. long, and if it had only had a length of 1 cm. the expansion would have been only 1-50th of that measured.

Hence the coefficient of linear expansion of brass is $\frac{0 \cdot 077}{82 \times 50} = 0 \cdot 000019$.

From this example it is easy to see that :—

$$\text{Coefficient of linear expansion} = \frac{\text{Measured Expansion}}{\text{Original length} \times \text{Degrees rise.}}$$

Experiment 48.

Determine the coefficient of expansion of a number of different metals.

The results may be compared with those given in the following table, which will also be useful for reference.

Coefficient of Linear Expansion.

Metal.	Coefficient. Degree Centigrade.
Aluminium	0·0000231
Brass	·0000188
Copper	·0000168
Iron, Cast	·0000107
Iron, Wrought	·0000125
Lead	·0000292
Nickel	·0000128
Platinum	·0000090
Silver	·0000192
Tin	·0000223
Zinc	·0000292

The Coefficient of Cubical Expansion.—In the case just considered we examined only the increase in *length* of the rod resulting from a rise of temperature. It is reasonable to suppose that the diameter increased also.

It occasionally happens that the increase in *volume* is required, and then we need the coefficient of *cubical* expansion of the body. This is the increase of unit volume when the temperature advances one degree.

Consider a cube whose edge is of unit length at a given temperature. If a is the coefficient of linear expansion of the material of which the cube is made, the length of the edge at a temperature one degree higher will be $(1+a)$.

Thus we see that :—

Volume of cube at initial temperature $=1^3$

,, ,, $1°$ higher $=(1+a)^3$.

But $(1+a)^3=1+3a+3a^2+a^3$.

The increase in volume is therefore $3a+3a^2+a^2$.

A reference to the table given above shows that the value of a is very small for all the metals mentioned. If any one is selected, and the value of $3a^2+a^3$ determined, it will be made

clear that these terms have so small a value as to be quite negligible.

Thus we may say that unit volume expands $3a$ units when the temperature is raised one degree. Hence the coefficient of cubical expansion is $3a$. But this is three times the value of the coefficient of *linear* expansion.

The Expansion of Liquids.—Just as normal work demands a consideration of the linear expansion of metals, so it requires data referring to the cubical expansion of liquids.

Experiment 49.

Determine the coefficient of cubical expansion of water. The density bottle described in Chapter III is suitable for this purpose. First clean and dry the bottle and weigh it. Then fill it with cold water at a known temperature and weigh again.

The whole is now immersed in a vessel of water in which a thermometer is suspended. This water is slowly raised to a temperature of 50° or 60° C., care being taken to keep the water in the vessel well stirred. In this way the water within the density bottle acquires the temperature of the outside water.

The bottle is now removed, the outside is dried, and when cool it is again weighed. An example should make the method clear.

Ex. 19. Find the coefficient of cubical expansion of water from the following data :—

Weight of empty density bottle, 9·842 grms.
,, ,, bottle full of water at 16° C., 34·839 grms.
,, ,, bottle and water after heating to 54°C. = 34·518 grms.

As the water in the bottle was heated it expanded, and some of it therefore flowed out through the small channel in the stopper. Thus the final weighing is less by an amount representing the weight of the expelled water.

Subtracting the weight of the empty bottle in each case we have :—

Weight of water at 16° C. = 24·997 grms.
,, ,, ,, 54° C. = 24·676 ,,
,, ,, ,, expelled by a rise of 38 degrees = 0·321 grms.

Following lines similar to those adopted in the case of the metal rod, we have :—

$$\left.\begin{array}{c}\text{Coefficient of}\\ \text{cubical expansion}\end{array}\right\} = \frac{\text{Volume expelled}}{\text{Volume remaining} \times \text{degrees rise.}}$$

Since the "volume expelled" and the "volume remaining" were at the same temperature (viz. 54° C.) these *volumes* are proportional to their *weights*. Hence we may write :—

$$\left.\begin{array}{c}\text{Coefficient of}\\ \text{cubical expansion}\end{array}\right\} = \frac{\text{Weight expelled}}{\text{Weight remaining} \times \text{degrees rise.}}$$

Substituting our values we get :—

$$\left.\begin{array}{c}\text{Coefficient of cubical expansion of}\\ \text{water between 16° C. and 54° C.}\end{array}\right\} \frac{0 \cdot 321}{24 \cdot 676 \times 38} = 0 \cdot 00034$$

It is necessary to add that water has a very irregular coefficient of expansion, and in the example given we have only determined the *average* coefficient over a given range of temperature. If another range of temperature had been employed a different result would have been obtained.

The following table will show how the coefficient of expansion of water varies with the temperature.

Temperature Range.	Coefficient of Cubical Expansion of Water.
0° to 10° C.	0·000014
10° ,, 20° C.	0·000147
20° ,, 30° C.	0·000255
30° ,, 40° C.	0·000377
40° ,, 50° C.	0·000421
50° ,, 60° C.	0·000489
60° ,, 70° C.	0·000556

Water is an exceptional substance in this respect, most other liquids being much more regular. The following table gives the coefficient of expansion of mercury over the same range of temperature.

Temperature.	Coefficient of Cubical Expansion of Mercury.
0°	0·00018170
20°	0·00018168
50°	0·00018183
70°	0·00018205

It will be seen that there is comparatively no variation in the coefficient, the actual variation being so small that it requires work of a very high degree of refinement to detect it.

A discerning student may have noticed that in Experiment 49 and Example 19, no reference was made to the fact that the glass vessel containing the water was also subject to the rise of temperature, and since the glass expanded the capacity of the vessel was increased.

The coefficient of expansion of the liquid determined in this way is therefore too low. It is called the *apparent* coefficient of expansion. When a correction has been applied for the expansion of the glass, we have the "*absolute*" coefficient of expansion. The coefficients given in the tables above are all absolute.

Of course the expansion of glass depends to some extent upon its composition, but the average coefficient of linear expansion of glass is about 0·000008.

Multiplying this by 3 we get the coefficient of cubical expansion, and it will be observed that it is very small indeed compared with that of water or mercury (or indeed, other liquids).

Hence the influence on the result is very small, and in addition we have the fact that in practical work liquids have to be held in a vessel, and it is the coefficient of apparent expansion which is mostly needed.

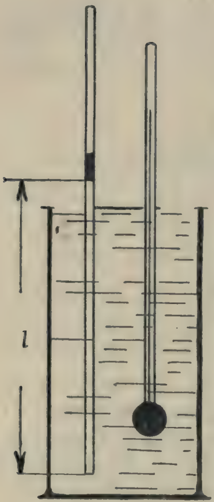

FIG. 88.

Experiment 50.

Expansion of Gases.—Obtain a piece of thin glass tube of not more than 2 mm. bore and about 30 or 40 cms. long. Draw a short thread of mercury into one end and seal up the other end in a Bunsen flame.

We now have a quantity of air enclosed in the tube by the mercury thread, the volume of the air being indicated by the length of the tube which it occupies.

The tube is supported in a large beaker of water which also contains a thermometer as shown in Fig. 88.

The water is slowly heated with constant stirring to keep its temperature uniform. The length (l) of the air column and the temperature of the water are recorded at frequent intervals.

We will again illustrate the method by an example.

Ex. 20. The following table gives corresponding readings of temperature and length of air column in Experiment 50. Find the coefficient of cubical expansion of air.

Temperature	16° C.	25° C.	35° C.	45° C.	55° C.
Length in cms.	24·5	25·25	26·2	27·0	28·0

A very casual glance at these numbers is sufficient to show that we are here dealing with expansion the magnitude of which is greatly in excess of anything hitherto experienced.

First express the results by means of a graph, which is shown in Fig. 89. The points lie very close about a straight line, and it will be observed that the graph has been produced backwards so as to include the temperature of 0° C.

If a little ice is available this point may be obtained experimentally, and should certainly be included if means permit.

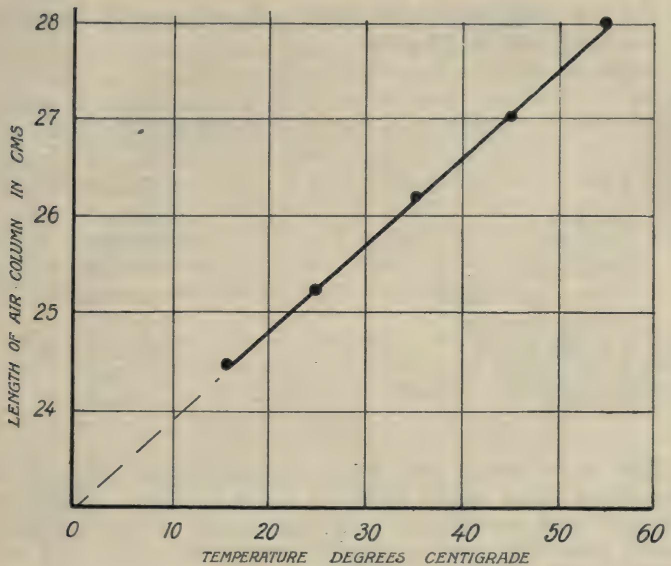

FIG. 89.

Selecting points on the graph we obtain :—

Volume at 0° C.	= 23·05 units.	
,, 50° C.	= 27·50	,,
Expansion for a rise of 50 degrees	= 4·45	,,

The coefficient of cubical expansion of a gas is given by :—

$$\frac{\text{Amount of expansion}}{\text{Volume at } 0^\circ \text{ C.} \times \text{degrees rise.}}$$

Hence we have :—

$$\text{Coefficient for air} = \frac{4{\cdot}45}{23{\cdot}05 \times 50.}$$
$$= 0{\cdot}0038.$$

Since gases expand so rapidly, it is necessary to state the temperature at which the volume is expressed in the above formula. 0° C. has been selected for this purpose.

The following table gives results of very careful measurements on a number of gases.

Gas.	Coefficient of cubical expansion between 0° C. and 100° C.
Air	0·003673
Oxygen	0·003676
Nitrogen	0·003673
Hydrogen	0·003661
Carbon monoxide	0·003669
Carbon dioxide	0·003710
Nitrous oxide	0·003762
Nitric oxide	0·003678
Methane	0·003690

This table brings out one remarkable fact. The gases mentioned have widely different characteristics. There are elements and compounds, a light gas such as hydrogen and heavy gases like carbon dioxide and nitric oxide, yet the coefficient of expansion is practically the same in every case.

It is found that some gases which are more readily liquefied by pressure deviate slightly from this apparently general rule. But sulphur dioxide, one of the worst offenders, has a coefficient of expansion of 0·00398, so the deviation is not very great.

The rule is so generally followed that it is commonly regarded as a " law " and goes under the name of Charles' law. It states that :—If the pressure remain constant, any given mass of gas expands one two hundred and seventy-third of its volume at 0° C. for each degree of rise in temperature.

Note that $1/273 = 0{\cdot}003663$, which is very close to the values given in the table above.

From this it is clear that if any gas has a volume of 273 units at 0° C., at 1° C. its volume would be 274 and so forth.

But these numbers represent the temperature on the absolute scale, and hence we may state Charles' Law in a much better form, thus :—

If the pressure remain constant, the volume of a given mass of gas varies directly as the absolute temperature.

Since the volume of all gases is affected to so large an extent by variations of temperature and pressure it is necessary to state all volumes under some agreed standard conditions of temperature and pressure, and not under the conditions under which a gas happens to be measured.

The standard conditions which have been agreed upon are a temperature of 0° C. and a pressure corresponding to a column of mercury 760 mm. high. These are denoted by N.T.P. or normal temperature and pressure.

Ex. 21. Some coal-gas was measured at a temperature of 18° C. under atmospheric pressure and found to have a volume of 875 c.c. The barometer stood at 752 mm. Reduce the measured volume to N.T.P.

Let V=volume at 0° C.

and v=volume at t° C.

The absolute temperature at 0° C.=273° A.

The absolute temperature at t° C.=$(273+t)$° A.

Now since the volume varies directly as the absolute temperature we have :—

$$\frac{V}{v} = \frac{273}{273+t}$$

$$\text{therefore } V = \frac{273v}{273+t}$$

Again, if v_1=volume of some gas when the pressure is p_1 and v_2=volume of the same gas when the pressure changes to p_2 by Boyle's law (see Chapter IV) we have :—

$$v_1\, p_1 = v_2\, p_2$$

$$\text{Hence } v_1 = \frac{v_2\, p_2}{p_1}$$

Now if p_1=760 mm. we get :—

$$v_1 = v_2 \frac{p_1}{760}$$

Applying this to the formula for correcting for temperature and denoting volume at N.T.P. by V and letting v=volume measured at $t°$ C. and p mm. pressure, we have :—

$$V = \frac{273v}{273+t} \times \frac{p}{760}$$

Substituting the values in the given problem we obtain :—

$$V = \frac{273 \times 875}{(273+18)} \times \frac{752}{760}$$

$$= \frac{273 \times 875 \times 752}{291 \times 760}$$

$$= 813 \text{ c.c.}$$

The Influence of Pressure.—When we considered the behaviour of a gas under pressure we took precautions to keep the temperature constant. In the more recent investigations of the effects of temperature change we have worked always under atmospheric pressure which, we have assumed, has not varied.

We will now consider the effect of keeping the *volume* constant while temperature and pressure change.

Suppose we have 273 c.c. of a gas at N.T.P. Keeping the pressure constant let us raise the temperature to 100° C. (*i.e.* 373° A.). The volume will now be 373 c.c.

Now keeping the temperature constant compress the gas back to its original volume. Let the pressure be p.

Boyle's law states that the product of pressure and volume is constant.

$$\text{Thus } 273 \times p = 373 \times 760$$

$$\text{Therefore } p = 760 \times \frac{373}{273}$$

$$= 1040 \text{ mm.}$$

This is directly proportional to the absolute temperature. Hence we may say :—

If the volume be kept constant, the pressure of a gas is directly proportional to the absolute temperature.

The Gas Equation.—Let V=volume of a gas at N.T.P. For a given mass of any given gas this will, of course, be constant.) Let v=the volume of this gas when the pressure is p and the absolute temperature is T.

It will be recalled that we obtained a formula for reducing volumes to N.T.P. in which—

$$V = \frac{273v}{273+t} \times \frac{p}{760}$$

Now $(273+t)$ is the absolute temperature at $t°$ C. and may be written T,

$$\text{Hence } V = \frac{273 \times v\,p}{T \times 760}$$

$$\text{Therefore } \frac{p\,v}{T} = \frac{760\ V}{273}$$

But V is a constant, and therefore the expression $\frac{760\ V}{273}$ is a constant. It is usually denoted by the letter R.

$$\text{Thus } \frac{p\,v}{T} = R,$$

$$\text{or } p\,v = RT.$$

This equation is known as the "Gas Equation." It is, however, only another form of the formula developed in Example 21.

Exercises 14.

1. A man is making a ring gauge and he is using $\frac{1}{4}''$ diameter cast steel check pins; their measured lengths at 62° F. are 8·0000″ and 8·0005″, if he holds the pins in his hands until the average temperature rises to 72° F., calculate the new length of the pins. Show by sketches some simple method of overcoming this alteration in length. The coefficient of expansion of cast steel is $1{\cdot}17 \times 10^{-5}$ per degree centigrade.

2. Repeat the above example, with pins 4·9995″ and 5·0000″ long and the same rise of temperature.

3. A number of standard cast steel check gauges of the following lengths, 2″, 4″, 6″, and 8″ have been certified as

correct at 62° F. to 0·00005″. What will be the lengths on a hot day if the temperature of the air is 95° F, ?

4. A cast steel cylinder is put into a furnace and heated from 30° C. to a temperature of 200° C. If the cylinder was originally 3·000″ diameter and 6·0000″ long, calculate its new length.

5. A 10″ diameter cast-iron water pipe is at a temperature of 56° F. The pipe 10 feet in length and the temperature falls to 32° F. What is the new length of the pipe ?

6. A hemispherical-ended boiler of mild steel is 4 feet in diameter and 22 feet in length. Determine the maximum alteration in length if the water it contains is heated from 62° F. to 212° F. The coefficient of expansion of mild steel is $1{\cdot}2 \times 10^{-5}$ per degree Centigrade.

7. A lathe is certified to have a mild steel leading screw whose error is not greater than +0·0005″ per foot at 62° F. What alteration in length will take place when the temperature alters to 85° F. ?

8. In a fluid gauge there is a mild steel rule, which is calibrated, and three inches represents 0·003″ or 1″ per 0·001″. In calibrating the instrument the following readings are taken :

Nominal scale reading	Corrections to be applied to the reading.
0″	−0·0000
0·001″	−0·0000
0·002″	−0·00005″
0·003″	−0·0001″

Give a list of corrections to be applied if the temperature of the scale is allowed to rise through 30°F. whilst the object measured is at the standard temperature.

9. A hard copper cylinder has to be 1·02″ diameter at 62° F. What alteration in diameter will take place if the cylinder is heated to 150° F. ?

10. A mild steel gauge measured 1·9850″ and a similar gauge for the bottom limit measured 1·9845″ when measured at 62° F. What alteration in length would you expect to

find if the man using these gauges held them in his hand until they reached a temperature of 89° F. ?

11. A horseshoe gauge has to measure 6·965″ inside the jaws and has been made accurate at 62° F. What will be the percentage error caused by an alteration in the temperature of the gauge of 20° F. ? Assume the gauge is mild steel and only the jaws are cast steel.

12. A tyre is to be put on a wheel and the wrought-iron hoop is made slightly less in internal diameter than the outside of the wheel. Give reasons for this and explain how the hoop is placed on the wheel.

13. A compound bar consists of two strips, one of brass and the other of mild steel, rivetted together. The bar is placed edgewise in a Bunsen flame, so that both metals are heated equally. Give sketches, showing how the metal bends when heated and cooled.

14. Why is a space left between the ends of railway lines ? Give sketches of the side-plates, bolts and bolt-holes which fasten the ends of the rails together.

15. A railway bridge is built up of joists and is 27 feet long. What alteration in length will take place if the length is 27 feet at a temperature of 62° F. and there are then temperature changes ranging from 32° F. to 95° F. ?

16. The compensated balance-wheel of a watch is made with the arcs of the wheel of metals of different coefficients of expansion. The metal with the smaller coefficient is the inner metal. Make a sketch and explain the action.

17. Explain, by the aid of sketches, six uses to which the property of the expansion of metals with heat is put.

18. Explain by the aid of sketches six common disadvantages which occur in practical working owing to the expansion of metals with heat.

19. Give reasons to show why it is advisable to have fire-bars in boilers loosely fitting.

20. Describe any experiment you have performed to determine the coefficient of expansion of metals.

21. The pattern-maker uses a special rule called the "contraction rule." For cast iron this rule is $\frac{1}{8}''$ per foot longer than the ordinary rule. Cast iron melts at 1,500° C. Assuming that the coefficient of contraction is constant throughout the cooling range (this is not true in practice) determine how much a rod of cast iron one foot long at the temperature given will contract. Let the final temperature be 35° C.

22. Describe in detail some method for keeping constant the rate of a clock. Variations in temperature must not have any effect on the length of the pendulum.

23. Explain what you mean when you say that 0·02 carbon steel has a mean coefficient of expansion of $11{\cdot}8 \times 10^{-6}$ between the temperatures of 15° and 200° C.

24. The distance between telephone posts is 80 yards and the wire used is copper. What difference in the length of the wire would you expect between 0° C. and 30° C.?

25. Convert the table given on page 164 from a table of linear expansion to a table of cubical expansion.

26. The coefficient of linear expansion of aluminium is 0·0000231. Determine the cubical expansion of a cube of aluminium of 3″ side if the temperature of the cube is raised 20° C.

27. Platinite is an alloy of iron with 42% nickel. This alloy has the same coefficient of expansion and contraction at atmospheric temperature as glass. Discuss the use of this alloy as a wire mesh for the manufacture of armoured glass. What difference would you expect if ordinary wire netting were used instead?

28. In the manufacture of electric lamps a small piece of platinum is fused into the glass. Why is platinum used in preference to cheaper metals?

29. Invar is an alloy of iron with 36% nickel and the coefficient of expansion with ordinary atmospheric changes of temperature is smaller than that of any other metal. The linear coefficient of expansion ranges from 0·000000374 to 0·00000044

for changes of 1° C. How does this compare with mild steel? Express the comparison as a vulgar fraction in its simplest form.

30. In surveying it is important that there should be as little alteration in length as possible in the steel tape used. Discuss the advantages of a steel tape 100 feet long made of invar as against one made of a low carbon steel.

31. The following table gives the relative volumes of water at different temperatures, compared with its volume at 4° C.

Degrees Centigrade—						
4	10	20	30	40	50	70
Volume—						
1·000	1·00025	1·00171	1·00425	1·00767	1·01180	1·02241

If the weight of one cubic foot of water at 4° C. is equal to 62·4245 lbs., plot a curve of the temperature and weight of one cubic foot of water.

32. An experiment was carried out on a brass rod to determine its coefficient of expansion and the following results were obtained :—

Length of the rod at 18° C. . .	50·084 cms.
Length of the rod at 98·5 C. . .	50·159 cms.

Determine the coefficient of linear expansion.

33. An experiment was carried out to determine the average coefficient of expansion of water. A bottle weighted with shot was weighed in air, the bottle was then weighed in water at a given temperature and then in water at a higher temperature. The following are the results :—

True weight of the bottle in air . . .	82·01 gram.
Weight of the bottle in water at a temp. of 10° C.	10·91 ,,
Weight of the bottle in water at 25° C. . .	11·05 ,,

Determine the average coefficient of expansion of water between the temperatures given.

34. An experiment, to determine the coefficient of expan-

sion of air at a constant pressure, was carried out and the following are the results :—

Length of air column at 17° C. = 24 cms.
" " " 100° C. = 31·8 cms.

Determine the average coefficient of expansion of air at a constant pressure.

35. An experiment was carried out by the aid of a Jolly's constant volume air thermometer, to find the coefficient of expansion of air at constant volume. The following are the results :—

Pressure of the atmosphere . .	752 millimetres.	
Temperature readings . .	17·4° C.	23·8° C.
Pressure readings in millimetres .	720	736

Determine the coefficient.

36. The following figures show how the volume of air alters with temperature in degrees Fahrenheit. Plot a graph showing how air expands with temperature.

Degrees F.—	32	40	50	60	70	80	90	100	110
Relative volumes—	1	1·021	1·043	1·066	1·089	1·110	1·132	1·152	1·173

37. From the following figures plot a graph showing the relation between volume of air and temperature :—

Temperature in degrees F.—	32	40	50	60	70	80	90
Volumes in cubic feet—	12·24	12·59	12·84	13·14	13·34	13·59	13·845

38. From the figures given, plot a graph to show how the weight of air varies with pressure at a constant temperature of 0° C.

Pressure by gauge in lbs.—	0	5	10	20	30	40	50	60
Weight in lbs. per cubic foot—	0·0864	0·116	0·1455	0·204	0·263	0·3215	0·380	0·4385

39. From the figures given, plot a graph to show how the weight of air varies with temperature, with a constant gauge pressure of 5 lbs. per sq. in.

Temperature of air in degrees F.—

—20	—10	0	20	30	40	50	60

Weight in lbs. per cubic foot—

0·1205	·1184	·1455	·1395	·1366	·1388	·1310	·1283

40. Plot a graph of barometric pressure in inches of mercury and altitude in feet.

Altitude in feet—

0	1000	2000	3000	4000	5000	6000	7000	8000

Barometric pressure in inches of mercury—

30	28·88	27·8	26·76	25·76	24·79	23·86	22·97	22·11

41. 150 cubic centimetres of air are measured at 30° C. If the temperature be raised to 60° C., determine the volume, assuming the pressure remains constant.

42. 150 cubic centimetres of air are measured at 30° C. and then cooled down to —30° C. By how much will the volume diminish ?

43. A long narrow tube was filled with mercury and inverted in a trough. At each experiment a little of the mercury was run out and the volume of the air measured. The pressure was determined in the usual way and the following results are given :—

Pressure in inches of mercury—

29·2	25	21	15	10

Volume as measured—

4·812	5·62	6·7	9·4	14

Plot a graph of pressure and volume and determine the value of pv.

CHAPTER XV

SPECIFIC HEAT

Quantity of Heat.—If a few cubic centimetres of water are placed in a test-tube and about a litre of water is put into a flask, on the application of a Bunsen flame to each for the space of one minute, a rise of temperature will result in both cases.

But whereas the small quantity of water is boiling, the larger quantity is still comparatively cold. Yet the flame had the same opportunity of imparting heat in both cases.

We see therefore that a high temperature does not necessarily indicate a large amount of heat. *Quantity* of heat, while depending upon temperature differences, depends upon other things too.

The Calorie.—The unit of quantity of heat generally adopted for scientific purposes is called the "calorie." It is defined as the quantity of heat which is required to raise one gramme of water through one degree Centigrade. Strictly speaking, it should be from 15° C. to 16° C.

This quantity is not materially affected by temperature, but in the definition it is desirable to state the interval of 15° to 16°, as the quantity of heat required to raise one gramme of water through one degree Centigrade is slightly different at other temperatures. The difference is, however, so slight as to be quite negligible except for very accurate work.

Where very large quantities of heat have to be dealt with, as in engineering work, it is desirable to have a larger unit and the "Grand Calorie" is employed. It is equal to 1000 calories.

The British Thermal Unit.—In technical work in this country the "British Thermal Unit" is commonly employed. It is the quantity of heat required to raise one pound of water through one degree Fahrenheit. It is often written as B.Th.U.

Since the variation due to temperature is so extremely small it is permissible to multiply a weight of water by its temperature change to express the number of units of heat gained or lost.

Experiment 51.

The Water Equivalent of a Vessel.—Obtain a thin metal canister with a capacity of about half a litre. Copper is a very suitable metal, but failing everything else a glass beaker *may* be used. Place it inside a larger vessel and pack the space between the two vessels with cotton wool.

A stirrer should be made by bending a piece of stiff copper wire into a suitable loop, and the addition of a thermometer completes the equipment.

This simple apparatus is called a calorimeter. Its use is best illustrated by considering an example.

Ex. 22. The inner vessel of a calorimeter was weighed and 64·98 grammes was the weight recorded. About 150 c.c. of cold water were added and the whole now weighed 215·76 grammes. The thermometer in the calorimeter registered 15° C. A quantity of heated water was added (whose temperature was 82° C.) and after stirring the temperature of the mixture was found to be 45° C. When cool the vessel and water were weighed, the whole weighing 344·12 grammes.

Subtracting the weight of the vessel from the first weighing we get :—

Weight of water at 15° C. = 150·78 grammes.

The weight of water added at 82° C. = 128·36 grammes.

Now the heat gained or lost by any quantity of water is the product of its weight and its temperature change.

Hence heat gained by
the cold water . . $= 150{\cdot}78 \times (45-15) = 4{,}523$ calories.
Heat lost by the hot water $= 128{\cdot}36 \times (82-45) = 4{,}749$ calories.

It will be observed that the hot water lost 226 more units of heat than were absorbed by the cold water, and since experiment has shown that energy in any form cannot be created out of nothing nor destroyed (in the sense of turning it into nothing), it follows that these 226 calories must be accounted for.

A little thought should make it clear that whereas the calorimeter, the thermometer and the stirrer were initially at the temperature of the cold water (viz. 15° C.) at the end of the experiment they were at the temperature of the mixture (viz. 45° C.). In other words, these things had been raised 30 degrees. It is reasonable to suppose that the 226 calories of heat were utilized for this purpose.

It is a very convenient practice to estimate how much additional cold water would have absorbed the same amount of heat, under the same conditions. Obviously $\frac{226}{30} = 7 \cdot 5$ grammes is the required amount.

This 7·5 grammes is called the " water equivalent " of the calorimeter. It means that in all subsequent experiments with this calorimeter, thermometer and stirrer, the heat which they absorb will be the same as that which 7·5 grammes of cold water would absorb.

Specific Heat.—We have already seen that the unit of heat is the amount of heat required to raise unit weight of *water* through unit temperature.

Now water is an unusually difficult substance to make warm. That is to say, most other substances are raised through a given range of temperature by the absorption of less heat than is required to raise an equal quantity of water through the same range of temperature.

The amount of heat which is required to raise unit weight of a substance through unit temperature is called the " specific heat " of that substance. We will now consider one or two methods by which the specific heat of a substance may be determined.

Ex. 23. A quantity of glycerine was placed in the calorimeter previously used (weight 64·98 grammes). The whole

then weighed 240·81 grammes. The thermometer and the stirrer were then placed in the glycerine, the former recording 17° C.

Hot water whose temperature was 76° C. was now added and the two liquids thoroughly mixed by stirring. The temperature of the mixture was 50·5° C. When cool the calorimeter and mixture were weighed and 373·22 grammes recorded.

Find the specific heat of the glycerine.

Collecting weights we have :—

Weight of the glycerine at 17° C. = 175·83 grammes.
Weight of water at 76° C. = 132·41 grammes.

Temperature through which glycerine and calorimeter are raised : 50·5 − 17 = 33·5 degrees.

Temperature through which the water falls : 76 − 50·5 = 25·5 degrees.

Heat gained by the calorimeter, etc.—

= Water equivalent × degrees rise.
= 7·5 grammes × 33·5 degrees.
= 251·2 calories.

Heat gained by the glycerine—

= Weight of glycerine × degrees rise × specific heat.
= 175·83 × 33·5 degrees × S.
= 5890 S calories,

where S = the specific heat of the glycerine.

Heat lost by the water—

= weight of water × degrees fall.
= 132·41 × 25·5.
= 3376·5 calories.

Now we know that :—

Heat gained = Heat lost.

Therefore $251{\cdot}2 + 5890\,S = 3376{\cdot}5$
Therefore $5890\,S = 3376{\cdot}5 - 251{\cdot}2$

$$\text{And } S = \frac{3125{\cdot}3}{5890}$$

$$= 0{\cdot}53$$

Experiment 52.

Find the specific heat of any liquid (which is not acted upon chemically by water) by the method illustrated in Example 23.

The Specific Heat of Metals.—For this purpose a known weight of the metal is usually heated to a known temperature and dropped into a calorimeter containing cold water.

It is very necessary for accurate work to provide some means of heating the metal so that it may be transferred from the heater to the calorimeter without loss of heat.

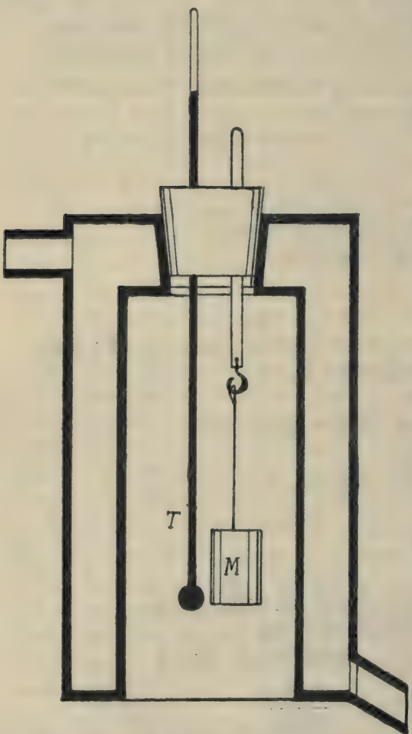

FIG. 90.

Fig. 90 shows a suitable form of heater. It consists of a cylindrical copper vessel provided with another cylinder inside so that there is an annular space through which steam may be passed.

An opening at the top is provided with a cork through which two holes are pierced. One of these carries a thermometer (T), and the other is closed with a short glass plug which secures a piece of thread on which the metal (M) is suspended so that it hangs near the bulb of the thermometer.

After steam has been passing through such a vessel for ten or fifteen minutes, the metal will be raised to a temperature

not far removed from 100° C., the exact temperature being recorded by the thermometer.

The calorimeter can now be held under the lower end of the heater and the glass plug having been removed, the metal may be gently lowered into the water contained by the calorimeter. The following example will indicate the method.

Ex. 24. A piece of brass weighing 157·24 grammes was placed in the heater just described. The calorimeter already employed (weighing 64·98 grammes and of water equivalent 7·5 grammes) was weighed with a quantity of cold water, the combined weight being 185·20 grammes. The thermometer being applied recorded 16·5° C.

The metal in the heater, having reached a temperature of 98° C., was lowered into the water, the temperature of which rose to 24·5° C.

Here we have :—157·24 grammes of brass falling through $98 - 24 \cdot 5 = 73 \cdot 5$ degrees.

And 120·22 grammes of water rising $24 \cdot 5 - 16 \cdot 5 = 8$ degrees.

Now heat gained by calorimeter, etc.—

$$= \text{Water equivalent} \times \text{degrees rise}$$
$$= 7 \cdot 5 \times 8 \text{ degrees}$$
$$= 60 \text{ calories.}$$

Heat gained by the water—

$$= \text{Weight} \times \text{degrees rise.}$$
$$= 120 \cdot 22 \times 8 \text{ degrees.}$$
$$= 961 \cdot 8 \text{ calories.}$$

Heat lost by the brass—

$$= \text{Weight of brass} \times \text{degrees fall} \times \text{specific heat.}$$
$$= 157 \cdot 24 \times 73 \cdot 5 \times S$$
$$= 11557 \ S \text{ calories.}$$

As before we may say that :—

$$\text{Heat gained} = \text{Heat lost}$$
$$\text{Hence } 60 + 961 \cdot 8 = 11557 \ S$$
$$\text{and } S = \frac{1021 \cdot 8}{11557}$$
$$= 0 \cdot 088.$$

Experiment 53.

Find the specific heat of a number of solids, including as many metals as are available.

Students of chemistry will find it of interest to plot a graph showing the relation between the specific heat of various metals and their atomic weights. A table should also be made giving the product of the atomic weight and specific heat of the metals he has experimented upon. This product is sometimes called the "atomic heat."

The specific heat of all substances varies with the temperature to a greater or less extent. The following table gives the specific heat of a number of common substances at the approximate working temperature of a laboratory. It will be useful for reference.

Substance.	Specific Heat.
Aluminium	0·217
Copper	0·093
Gold	0·031
Iron	0·109
Lead	0·031
Magnesium	0·248
Mercury	0·033
Nickel	0·109
Platinum	0·032
Silver	0·056
Tin	0·054
Tungsten	0·034
Zinc	0·092
Glass	0·2
Porcelain	0·26
Silica	0·17
Acetic acid	0·47
Ether	0·55
Ethyl alcohol	0·57

The Specific Heat of a Gas.—The general principle underlying the method of determining the specific heat of a gas is the same as that already described in the case of solids or liquids. The technical details of the apparatus are, however, much more complex, as a gas occupies such a large volume compared with its weight.

Just as in the case of other substances the specific heat of a gas depends to some extent on the temperature, but—and this is not the case with other substances—the pressure here plays an important part.

The following table gives the specific heat of a few common gases at the average temperature and pressure of the atmosphere.

Substance.	Specific Heat.
Air	0·2408
Carbon dioxide	0·202
Hydrogen	3·4
Nitrogen	0·242
Oxygen	0·218

It may be of interest to add that the specific heat of steam, at atmospheric pressure and a temperature of 100° C., is 0·435.

In all these cases it is assumed that the pressure has remained constant. That is, as the gas is heated it expands; at least, it will do so unless it is prevented by applying pressure. In that case, however, the pressure would not be constant.

We have already seen that the atmosphere exerts a pressure of about 15 pounds per square inch, and if a gas expands under this pressure it has, so to speak, to push this pressure back to make room for itself. But this requires energy, and doubtless some of the energy of the heat has to be applied to this purpose.

Hence when any gas is heated under constant pressure some of the heat goes to make the gas warm and some of it goes to supply the energy required to make room for the expansion.

Of course this is true also of solids and liquids, but in these cases the expansion is so small that the energy thus absorbed is entirely negligible.

From what has been said, it will be seen that if a gas is so enclosed that it cannot expand, that is, if its *volume* is kept constant, the specific heat will be less.

Although it is possible directly to measure the specific heat at constant volume, it is often more practicable to determine the ratio of the specific heat at constant pressure to that at constant volume.

The following table gives an indication of the quantities in question.

Substance.	Specific Heat at Constant Pressure.	Specific Heat at Constant Volume.	Ratio.
Air	0·24	0·171	1·404
Bromine vapour	0·057	0·044	1·29
Carbon dioxide	0·202	0·153	1·319
Hydrogen	0·341	0·242	1·408
Oxygen	0·218	0·156	1·398

The following table should also be studied :—

Substance.	Ratio of Specific Heat at Constant Pressure to that at Constant Volume.
Mercury vapour	1·67
Helium	1·63
Hydrogen	1·408
Air	1·404
Oxygen	1·398
Steam	1·32
Carbon dioxide	1·319
Bromine vapour	1·29
Ethane	1·182
Ether vapour	1·09

Students of chemistry will notice that the ratio is greater in the case of elements than in the case of compounds.

Experiment has shown that the ratio depends on the constitution of the molecule, becoming less as the molecule becomes more complex.

Isothermal Expansion.—It will be recalled that when Boyle's law was the subject of investigation in Chapter IV the gas was expanded (or compressed) very slowly in order to maintain a constant temperature. Such a process is called "isothermal" expansion. For a gas to expand isothermally, time and facilities must be given for the heat to flow *into* the gas in order to keep the temperature up.

On the other hand, if it is desired to compress the gas isothermally, time and facilities must be provided for heat to flow *out of* the gas.

Adiabatic Expansion.—If a gas is expanded (or compressed) with extreme rapidity there is no time for heat to flow into (or out of) the gas. In such cases the expanded gas becomes colder, or, if compression has taken place, the gas has become hotter.

This is called "adiabatic" expansion (or compression). Few experimental processes are sufficiently rapid to get perfect adiabatic expansion, but the process is approximately approached in the expansion of the working fluid in the cylinders of some high-speed engines.

Exercises 15.

1. Determine the number of calories and grand calories in the following cases :—

1. 20 lbs. water,	Rise in temp. in degrees Centigrade	30
2. 56 lbs. water	,, ,, ,, ,,	54
3. 37·5 lbs. water	,, ,, ,, ,,	85
4. 42·5 lbs. water	,, ,, ,, ,,	100.

2. In the above examples determine the number of B.Th.U. of heat given to the water.

3. Define the "Grand Calorie," the "Calorie," and the "British Thermal Unit."

4. Why is water the best substance to use in a hot-water bottle? Contrast the use of water with that of powdered silica and any other substance you think suitable.

5. The following results refer to a Diesel engine. The weight of cooling water in lbs. per brake horse-power used per hour and the rise in temperature in degrees Fahrenheit are given.

Determine the B.Th.U. per brake horse power per hour carried away by the cooling water.

Cooling water used in lbs. per B.H.P.	145	18·6	45
Rise in temp.	29·2°F.	118°F.	55·3°F.

6. In an engine test the total cooling water used in lbs. per hour was 115·65, the inlet temperature of the water was 54° F. and the outlet temperature was 103° F. Determine the B.Th.U. of heat carried away by the cooling water per hour.

7. The heat supply of an engine in B.Th.U. per brake horse power per hour was, (*a*) 14200, (*b*) 12000, (*c*) 11000, (*d*) 10200. What are these results in B.Th.U. per B.H.P. per second. Give these results in calories per second.

8. Professor Robinson gives the following calorific values.

Oil.	Calorific value in B.Th.U. per lb.
Refined Royal Daylight	20286
Refined petroleum	19885
Double refined	19955
Pratt's motor spirit	18610

Give these calorific values in calories per gramme.

9. Judge, "Handbook for Modern Aeronautics," gives the estimated fuel consumption in pints per hour for the following types of engines :—

Curtis *OX*5 . .	76	pints per hour at 6000 feet.		
ABC Dragon Fly .	256	,,	,,	,,
B.R.1 . .	129	,,	,,	,,
B.R.2 . .	196·8	,,	,,	,,

Assuming that petrol of 0·68 specific gravity and having a heat value of 19200 B.Th.U. per lb. was used; determine the B.Th.U. used per hour.

10. Determine the water equivalent of 0·75 lbs. of each of the following metals:—Aluminium, copper, iron, lead, magnesium, tin, and zinc.

11. Explain what is meant by "specific heat."

12. A gas ring weighs 3 pounds and is at 62° F. After boiling water its temperature is 150° F. What is the water equivalent of the ring and how many B.Th.U. has it absorbed?

13. In an experiment to determine the mechanical equivalent of heat the following results were obtained:—

Water in the drum, 300 grammes.

Water equivalent of the drum and thermometer, 40·75 grms.

Original temperature of the water, 15·92 C.

Final temperature of the water, 16·95 C.

Determine the calories of heat generated.

14. An experiment was carried out to determine the specific heat of iron tacks. The following are the results:—

Weight of the copper calorimeter, 67·135 grammes.

Weight of calorimeter and water, 101·43 grammes.

Original temperature of the water, 13° C.

Temperature of the tacks before dropping into the water, 97° C.

Temperature of the water and tacks after mixing, 16·2° C.

Weight of the calorimeter, water and tacks, 111·75 grammes.

Determine the specific heat of the tacks.

15. An experiment was carried out to determine the water equivalent of a copper calorimeter.

Weight of the calorimeter, 107·65 grammes.

Initial temperature of the calorimeter, 13° C.

Temperature of warm water before placing in the calorimeter, 41·3° C.

Final temperature of water and calorimeter, 39° C.

Weight of the calorimeter and the water, 236·22 grammes.

Determine the water equivalent from these results and also by assuming the specific heat of copper.

16. An experiment was carried out to determine the specific heat of copper and the results are as follows :—

Weight of the copper cylinder in grammes, 93·71.
Weight of the inner cylinder or calorimeter, 107·65 grammes.
Weight of the calorimeter plus added water, 292·5 grammes.
Initial temperature of the water in the calorimeter, 12·5° C.
Temperature of the copper cylinder, 99·7° C.
Final temperature of the water and the copper cylinder, 16·5° C.

Determine the specific heat of the copper cylinder.

17. To take into account the variation of specific heat with temperature, the specific heat of copper may be written as follows :—

Specific heat of copper at t° C., $=0{\cdot}0917+0{\cdot}000048t$.
Calculate the specific heat of copper for the following temperatures :—50, 100, 150, 200, 250, and 300° C. Plot a graph of temperature and specific heat.

18. A glass vessel at 30° C. weighs 980 grammes. If 2000 grammes of water at 40° C. are poured into the glass vessel, determine the final temperature of the water. Take the specific heat of the glass as 0·12.

19. Determine the B.Th.U. of heat carried away per hour by the cooling water in a 50 I.H.P. gas engine. Use the following formula :—

$$w\,(t - t^1) = 0{\cdot}30 \times \text{I.H.P.} \times \frac{33000}{778} \times 60.$$

Where w = lbs. of water per hour.
t = final temperature Fahrenheit.
t^1 = initial temperature Fahrenheit.
I.H.P. = indicated horse power.

20. In a gas engine the heat distribution is as follows :—

Work as indicated horse power . .	27·1 %
Carried away by the jacket water . .	49·5 %
Carried away by the exhaust gases . .	23·4 %
Total	100 %

If the heating value of the fuel is 1041 B.Th.U. per cubic foot, show how the heat in each cubic foot of gas is distributed.

21. Determine the thermal capacity per degree Centigrade of a line of hot water pipe, 3″ internal diameter, 4″ external diameter, 30 yards long and made of cast iron. Neglect the flanges.

22. The following results for an economizer are given :—

Temperature of the water as it enters the economizer.	Temperature of the water as it leaves the economizer.
84·2° F.	196·2° F.
40° F.	185·4° F.
101° F.	237·0° F.

Determine how many B.Th.U. per 100 lbs. of water are given up as it passes through the economizer.

23. The following table shows pressures and volumes in a single stage compressor. Column 2 gives volumes with constant temperature or isothermal compression, column 3 volumes as obtained in actual compressors, whilst column 4 gives volumes for adiabatic compression. Plot three curves showing pressure as abscissae and the three different volumes as ordinates. Also prove that *pv* in every case of isothermal compression gives a constant :—

Pressure absolute.	Volume at constant temp.	Volume.	Volume.
14·7	1	1	1
15·7	0·9363	0·948	0·954
16·7	0·8803	0·903	0·910
17·7	0·8305	0·862	0·876
18·7	0·7862	0·825	0·841
24·7	0·5952	0·660	0·692
29·7	0·4950	0·570	0·607
34·7	0·4237	0·503	0·544

24. If the total volume of a cylinder containing air at an absolute pressure of 14·7 lbs. per square inch is 12 cubic feet, what will the pressure become when the piston has moved

N

so as to reduce this volume to 5 cubic feet, the compression being isothermal ?

25. With isothermal compression $pv = C$. Plot graphs of pressure and volume for the following gases :—air, oxygen and nitrogen. Let the gas be at 14·7 lbs. per sq. in. pressure and take pressure increasing by 5 lbs. to 34·7.

Atmospheric air at 14·7 lbs. per sq. in. and 62° F. occupies a volume of 13·14 cubic feet for 1 lb.

Oxygen as above 11·88 and nitrogen 13·54.

CHAPTER XVI

LATENT HEAT

Experiment 54.

Change of State.—Arrange the apparatus shown in Fig. 84 so that the flask is about half full of cold water, and the thermometer has its bulb just below the surface of the water.

Place a small flame of a Bunsen burner beneath the flask, and read the thermometer at regular intervals, say every half minute.

Commencing with the water cold, continue the readings for a few minutes after the water has boiled.

Plot a graph showing the relation between the time and the temperature.

Now a consideration of this graph will show us that at first the rate at which the water gains heat is fairly uniform, and if the weight of the water is known it is possible to state the number of calories per minute which are absorbed by the water.

Near the boiling point, however, the rate of absorption appears to fall off, and when the water actually begins to boil the temperature remains constant.

Now it is unreasonable to suppose that the burner ceased to supply heat at the moment when the water boiled. Yet if the heat is supplied we must decide on its destination.

Latent Heat.—The behaviour of the water is explained thus: At first the heat supplied to the water was employed in increasing its temperature. This heat is called "sensible heat," because it is heat which is recognized by our sense of touch.

When the water commenced to boil the heat which passed

into it was used to supply the energy required to change the state of the water from the liquid to the gaseous state. This heat is called "latent heat."

Let us think for a moment how events would be affected if no latent heat were required. Suppose a pint of cold water were put into a kettle and the latter set upon a fire. The water would gradually get hot and ultimately the boiling point would be reached. At this moment the whole of the water would instantaneously pass into steam, which would occupy a space of approximately 1,700 pints. In other words, a very violent explosion would occur.

Fortunately this is not the case. When the water reaches the boiling point it passes into steam at a rate which is determined by the supply of the necessary latent heat.

The number of units of heat which are required to convert unit weight of water at the boiling point into steam at the same temperature is called the "latent heat of evaporation of water." Very often it is referred to as the "latent heat of steam."

The method of determining this quantity is again best illustrated by taking an example.

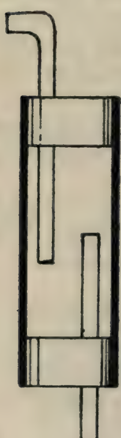

FIG. 91.

Ex. 25. Using the calorimeter, etc., of previous experiments (weight 64·98 grms. and water equivalent 7·5 grms.) a quantity of cold water is introduced. It now weighs 270·10 grms. The temperature of this water is 15·5° C.

Steam is now passed into the water by means of a glass tube which emerges from a "steam trap," which is shown in Fig. 91. This simple appliance serves to remove the particles of water formed by the steam condensing during its passage from the boiler to the calorimeter, and thus only "dry" steam (as it is called) enters the calorimeter.

When the passage of steam was stopped the thermometer in the calorimeter recorded $20\frac{1}{4}$° C. On cooling, the calorimeter and water weighed 271·74 grms., the increase in weight representing the weight of steam which had been condensed.

Collecting the data we have :—

Weight of water at $15\frac{1}{2}°$ C. $=205·12$ grms.
Weight of steam condensed $=1·64$ grms.
Rise in temperature of water and calorimeter $20\frac{1}{4}-15\frac{1}{2}=4\frac{3}{4}$ degrees.
Fall in temperature of condensed steam $100-20\frac{1}{4}=79\frac{3}{4}$ degrees.

Now heat gained by calorimeter, etc. = Water equivalent $\times$ degrees rise.

$7·5$ grms. $\times 4\frac{3}{4}$ degrees.
$=35·6$ calories.

Heat gained by water :—

$=$ weight of water $\times$ degrees rise.
$=205·12$ grms. $\times 4\frac{3}{4}$ degrees.
$=974·3$ calories.

Heat lost by steam in condensing to water at 100° C. :—

$=1·64$ grms. $\times L$.
$=1·64L$ calories.

where L is the latent heat.

Heat lost by this condensed steam in falling from 100° C. to $20\frac{1}{4}°$ C. :—

$=1·64$ grms. $\times 79\frac{3}{4}$ degrees.
$=130·5$ calories.

Equating heat gained and heat lost we have :—

$35·6+974·3=1·64L+130·5.$
$\therefore\ 1·64L=879·4.$
And $L=536$ calories per gramme.

Latent heats may be expressed in calories per gramme or B.Th.U.'s per lb. Since the gramme and the lb. enter into the unit of heat it is only the temperature scale which affects latent heats. That is, a latent heat in the latter units is $\frac{9}{5}$ of that in the former units.

The best results give the latent heat of vaporisation of water as 537 calories per gramme or 967 B.Th.U.'s per lb. The latent heat of vaporisation of ether is 91·3 and of ethyl alcohol 206 calories per gramme. These values all refer to vaporisation at atmospheric pressure.

Experiment 55.

Find the latent heat of evaporation of water by the method described in Example 25.

Latent Heat of Fusion.—Just as it needs heat to convert water into steam after it has been raised to the boiling point, so it needs heat to melt a solid after it has been raised to the melting point. The number of units of heat which are necessary to convert unit weight of a solid into a liquid at the melting point is called the "latent heat of fusion" of that solid.

Ex. 26.—The calorimeter, weighing 64·98 grms. (of water equivalent 7·5) had some water placed in it and its weight recorded. The combined weight was 192·39 grms., the temperature being 18° C.

A small lump of ice was carefully dried by means of blotting-paper and dropped into the calorimeter. The water was stirred until the ice melted, and its temperature was then 12° C. The weight was now 201·8 grms.

To find the latent heat of fusion of ice, first collect data as follows :—

Weight of water at 18° C. = 127·41 grms.
Weight of ice = 9·41 grms.

Rise of temperature of melted ice = 12 degrees.
Fall of temperature of water = (18 − 12) = 6 degrees.

Heat gained by ice in melting and thus changing from ice at 0° C. to water at 0° C. :—

= Weight of ice × latent heat.
= 9·41 grms. × L.
= 9·41 L calories.

Heat gained by melted ice in rising from 0° C. to 12° C. :—

= Weight of ice × degrees rise.
= 9·41 grms. × 6 degrees.
= 56·5 calories.

Heat lost by calorimeter, etc. :—

$=$Water equivalent$\times$degrees fall.
$=7{\cdot}5$ grms.$\times 6$ degrees.
$=45$ calories.

Heat lost by water :—

$=$Weight of water$\times$degrees fall.
$=127{\cdot}41$ grms.$\times 6$ degrees.
$=764{\cdot}5$ calories.

Now heat gained$=$heat lost.

$\therefore\ 9{\cdot}41L+56{\cdot}5=45+764{\cdot}5.$
$9{\cdot}41L=753.$
$\therefore\ L=80$ calories per gramme.

The best experiments give a value for the latent heat of fusion of ice of 79·7 calories per gramme, which is equivalent to 143·3 B.Th.U. per lb.

Experiment 56.

Find the latent heat of fusion of ice by the method described in Example 26.

In cold weather it may be desirable slightly to heat the water in the calorimeter. It should be done after weighing, and the temperature to which it is raised should not be over 20° C.

It will be observed that the temperature of the ice is assumed to be 0° C. This will be so unless the room in which the experiment is conducted is at a temperature well below the freezing point. This is not likely to be the case.

Any solid which can be melted has, of course, a latent heat of fusion. The following table gives the melting point and the latent heat of fusion of a number of common metals.

The method adopted in the determination of these latent heats varies with the nature of the metal. In no case, however, does the determination form a suitable experiment for the student at this stage of his work.

Metal.	Melting Point. Degrees Centigrade.	Latent Heat of Fusion in Calories per Gramme.
Aluminium . . .	625	80
Bismuth . . .	268	12·6
Cadmium . . .	320	13·7
Copper . . .	1050	43
Iron	1130	59
Lead	326	5·9
Mercury . . .	$-38{\cdot}7$	2·77
Nickel	1480	4·6
Platinum . . .	1775	27
Silver	962	25
Tin	231	14·6
Zinc	433	28

Change of Volume on Melting.—Most solids when they melt increase in volume. Ice is an exception to this rule. Indeed, water is an abnormal substance altogether in this respect.

Consider a quantity of water at, say, 15° C. If this is cooled it contracts until a temperature of 4° C. is reached, when water reaches its maximum density. From 4° C. to 0° C. the water expands as it cools. On turning into ice a considerable expansion occurs, 1 c.c. of water at 0° C. becoming 1·09 c.c. of ice at 0° C.

One of the effects of this abnormal behaviour is the occurrence of burst water-pipes in frosty weather. As the water within the pipe turns into ice it expands to the extent of 9 per cent. of its volume.

This expansion is absolutely necessary if the ice is to be formed, and a gigantic pressure is necessary to keep water liquid below 0° C.

Ordinary water-pipes cannot withstand this pressure, and consequently burst. Very often the fact is not made evident at once because the water has now become solid ice. When the temperature rises, however, the ice melts and a serious leakage indicates the burst pipe.

Just as great pressure applied to water will prevent its freezing, so a similar pressure applied to ice will melt it.

If a large block of ice be supported on two trestles as shown in Fig. 92, a steel wire may be passed over it and the free ends joined underneath so as to support a heavy weight. (A half hundred-weight forms a suitable load.)

The pressure of the wire on the upper surface of the ice is usually sufficient to melt the ice immediately beneath it, and the wire of course sinks into the narrow channel of water which it produces.

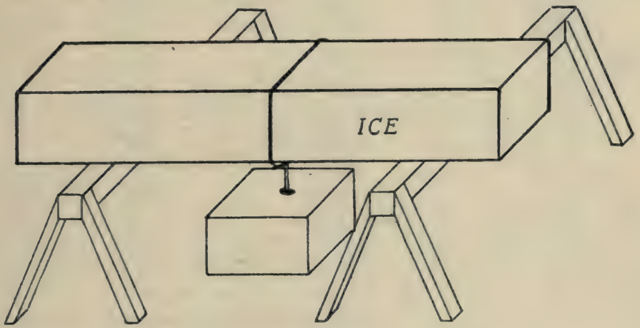

FIG. 92.

But ice cannot melt without the absorption of its latent heat, and since this is not forthcoming from external sources it has to supply the heat itself. Hence the water which is produced is at a temperature well below the freezing point.

Since this water is now above the wire it is no longer subjected to pressure, and it therefore immediately returns to the solid state, that is, ice.

The process is continued, and in time the wire will pass entirely through the block of ice without cutting it into two pieces.

Freezing Water by Boiling.—We have already seen that the boiling point of water depends upon the pressure to which it

is subjected. The following tables show the boiling point of water at various pressures, commencing with the normal atmospheric pressure in each case.

Pressures below one Atmosphere.		Boiling Point of Water.
In mm. of mercury.	In lbs. per sq. inch.	
760	14·7	100° C.
525·8	10·15	90° C.
355·1	6·85	80° C.
233·5	4·50	70° C.
149·2	2·88	60° C.
92·3	1·78	50° C.
55·1	1·06	40° C.
31·1	0·60	30° C.
17·5	0·34	20° C.
9·2	0·178	10° C.
4·6	0·089	0° C.

Pressures above one Atmosphere.		Boiling Point of Water.
In mm. of mercury.	In lbs. per sq. inch.	
760	14·7	100° C.
1074·5	20·7	110° C.
1489	28·8	120° C.
2026	39·1	130° C.
2709·5	52·3	140° C.
3569	68·9	150° C.
4633	89·2	160° C.
5937	114·7	170° C.
7514	145·2	180° C.
9404	181·8	190° C.
11647	225	200° C.

Experiment 57.

Obtain a spherical flask and fit it with a rubber stopper. Half fill the flask with water and boil it for a few minutes. After a time the air which occupied the space in the flask above the water will have been expelled and replaced by steam.

Now remove the flame and insert the stopper. Invert the flask and support it in a stand as shown in Fig. 93.

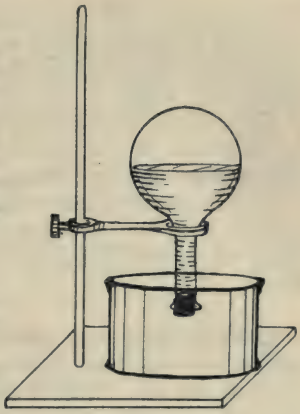

Fig. 93.

By this time the water will have cooled somewhat below 100° C., yet if *cold* water be poured over the flask, the water inside the flask will boil vigorously.

This is due to the cold water condensing the steam which occupies the space above the water. The condensation, of course, reduces the pressure, and although the water was not hot enough to boil under the higher pressure, it is able to do so when the pressure is reduced.

Experiment 58.

If an air-pump, such as that shown in Fig. 21, is available, it may be used for the purpose of reducing the pressure instead of the method just described.

Support a thermometer in a vessel of warm water and place the whole under the bell-jar of an air-pump.

As the pump is operated the pressure falls and the water ultimately commences to boil. The boiling will continue if the pressure is kept sufficiently low.

The thermometer will show, however, that as the water *boils* its temperature *falls*.

The reason is similar to that which caused the ice to melt under pressure and to revert to ice when the pressure was removed.

In this case we have water being made to boil without

supplying it with the necessary latent heat. Hence every gramme of water which is converted into steam under these conditions has to take over 500 calories of heat from the remaining water.

This, of course, causes a considerable fall of temperature. In addition it makes it necessary to reduce the pressure still further if the boiling is to continue.

It will readily be seen that if the pump were approximately perfect it would be possible to reduce the temperature to such an extent that the remaining water would begin to freeze.

Another Effect of Latent Heat.—Most bathers have experienced conditions when it has felt warmer in the water than in the air. This is often the case when a wind is blowing. Yet a thermometer frequently shows that the reverse is actually the case.

When the surface of one's body is beneath the water, it "feels" the temperature of the water. When it is exposed to the air, however, the film of water commences to evaporate (especially if there is a "dry" wind), and the latent heat which is necessary for this vaporisation is taken from one's body and produces the sensation of cold.

It will be seen that the actual temperature of the air has very little influence on this sensation.

In certain hot countries (*e.g.* Portugal) coarse earthenware jars are used to keep drinking-water cool. These jars are very porous, and when filled with water some of it soaks through to the outside.

This film of water readily evaporates in the dry air, and the latent heat (or at any rate a large proportion of it) is absorbed from the water within the jar. Thus the latter is kept cool.

It has been said that it is not "hot enough" in this country to use this method of cooling water. It would probably be more correct to say that the air is not often sufficiently *dry*.

Exercises 16.

1. Describe in detail a method of determining the latent heat of steam. Point out the principal sources of error in the experiment and the precautions to be taken to minimize them.

2. In the above experiment state how the results are affected by using steam that is not "dry."

3. Suppose that you have a small quantity of ice at the melting temperature, and that you gradually melt it. What changes of temperature and volume does the ice undergo?

4. Describe fully how you determine the melting point of paraffin. What are the changes that you observe during the change of state?

5. State how you would determine the latent heat of paraffin.

6. An experiment was carried out to determine the latent heat of steam and the following were the results obtained:—

Weight of copper calorimeter in grammes, 82·5.

Weight of water used for condensing the steam, 98·4 grammes.

Initial temperature of the above water, 17·3° C.

Maximum temperature of the water after condensing the steam 58° C.

Weight of the condensed steam, 7·4 grammes.

Determine the latent heat of steam in calories per gramme and B.Th.U. per lb.

7. An experiment was carried out to determine the latent heat of ice and the following are the results:—

Weight of the copper calorimeter, 105·87 grammes.

Weight of the calorimeter and the water, 274·7 grammes.

Original temperature of the water in the calorimeter, 30° C.

Final temperature of the water after mixing well with the dry ice, 20° C.

Weight of the water, ice, and calorimeter, 292·7 grammes.

Determine the latent heat of ice in calories per gramme.

8. The results of an experiment are given which was carried out to determine the latent heat of steam :—

Weight of calorimeter, 107·1 grammes.

Weight of the thermometer, 128·35 grammes.

Weight of calorimeter, thermometer, and water 323·9 grammes.

Temperature of the water in the calorimeter, 16·7° C.

Final temperature of the water after condensing the steam, 41° C.

Weight of the water, calorimeter, thermometer, and the steam, 332·4 grammes.

Determine the latent heat in calories per gramme.

9. The latent heat of steam varies with the pressure, and may be calculated from the following formula :—

$$\text{Latent heat} = 1114 - 0{\cdot}7t$$

where t is the temperature in degrees Fahrenheit of the boiling point of water at the given pressure. Calculate the latent heat for the following temperatures: 212, 222, 232, 242, 252, and 262° F. Plot a graph of latent heat and temperature. What are the units in which the latent heat is expressed ?

10. The total heat of steam includes the *sensible* and the *latent* heat and may be calculated from the formula :—

$$H \text{ (total heat)} = 1082 + 0{\cdot}305\ t^{\circ}\ \text{F.}$$

Calculate the total heat of steam for the following temperatures :—212, 220, 230, 240, 250 and 260° F. Plot a graph of H and t.

11. Watt, the great inventor and engineer, has given a full account of the early experiments he carried out which led up to his first patent. Steam was carried from a kettle by means of a piece of glass tubing to a cylindrical glass jar containing water. At the completion of the test the water in the glass vessel was then found to have increased about $\frac{1}{6}$ part from the condensed steam. Consequently, water converted into steam can heat about 6 times its own weight of well water to 212°, or till it can condense no more steam.

Watt had to ask Dr. Black to explain this. Give your explanation.

12. In a lecture by Geo. Babcock at Cornell University on "The Theory of Steam Making," the following occurs :— "It follows that if we could reduce steam at atmospheric pressure to water, without loss of heat, the heat stored within it would cause the water to be red hot; and if we could, further, change it to a solid, like ice, without loss of heat, the solid would be white-hot, or hotter than melted steel, it being assumed that the specific heat of water remained normal." Explain this.

13. The following also occurs in the same lecture :—"The heat which has been absorbed by one pound of water to convert it into one pound of steam at atmospheric pressure is sufficient to have melted 3 pounds of steel or 13 pounds of gold." Taking the necessary figures from the tables given prove the truth of this statement.

14. If some volatile liquid is placed on the hand, the hand feels cold. Explain this.

15. A person has a headache and places a handkerchief with eau de Cologne on it across his forehead. Give reasons for this.

16. Experiments were carried out to determine the efficiency of small oil furnaces (see article by the authors, *English Mechanic*, June, 1912), and the following are some results :—

Material.	Weight of metal melted.	Weight of petroleum used.
Cast iron	5 lbs.	4 lbs.
Aluminium	2	1·2
Lead	4	0·416

Taking the petroleum as having a calorific value of 18,500 B.Th.U per lb., determine the B.Th.U. used in melting the metal. Determine the B.Th.U. required to melt each batch of metal. If the thermal efficiency is the B.Th.U. required to melt the metal divided by the B.Th.U. used, determine the thermal efficiency in each case.

17. A test was carried out in a cupola and 1,232 lbs. of coke were required to melt 20,160 lbs. of cast-iron. Determine the total heat required to melt this quantity of metal, the total heat in the coke and the thermal efficiency. The calorific value of coke is 13,000 B.Th.U. per lb.

18. The table here given shows how the sensible and latent heats of steam vary with the pressure :—

Pressure in lbs. per sq. in.	Temperature F.	Latent heat.
1	101·99	1043·0
2	126·27	1026·1
5	162·34	1000·8
10	193·25	979·0
15	213·03	965·1
20	227·95	954·6
30	250·27	938·9
40	267·13	927

Plot a graph of pressure and temperature of boiling (column 2) and a graph of pressure and latent heat.

19. What is meant by the "boiling point" of a liquid? How is it affected by change of pressure?

20. Describe any form of boiler you have seen and state how the pressure is kept from rising to a dangerous amount.

21. Give a sketch of a domestic hot-water supply system and explain how the pressure is prevented from rising.

22. The following statement is given in a catalogue :— "Well designed boilers under successful operation will evaporate from 7 to 10 pounds of water per pound of first-class coal."

Suppose that the boiler is working at 150 lbs. per sq. in. and the total heat given to the steam per lb. of water is 1,191·2 B.Th.U., whilst the calorific value of the coal is 13,800 B.Th.U., determine the thermal efficiency for the two cases given.

23. Ten pounds of water are enclosed in a small steam boiler, the temperature of the water is 68° F. How much

heat must be given to the water to generate 8 pounds of steam at atmospheric pressure?

24. Twelve pounds of steam at 100° C. are blown into a tank containing 300 lbs. of water at 40° C. Find the resulting temperature.

CHAPTER XVII

CONDUCTION, CONVECTION, AND RADIATION

Conduction.—We have frequently mentioned the fact that heat "flows" from one body to another according to temperature differences. In this chapter we will consider the *methods* by which heat can pass from one body to another.

Experiment 59.

Obtain a few pieces of thick wire of different metals, but of the same dimensions, about 5 or 6 inches long. Holding each specimen by one end, apply the other end to the Bunsen flame.

It will be observed that the heat absorbed from the flame travels along the wire and ultimately reaches the fingers.

The student should find no difficulty in discerning between "good conductors" like copper and those metals which convey heat less readily.

The property which all forms of matter possess in a greater or less degree, which enables the individual particles to pass heat from one to another, is known as "conduction."

Of course, there are good conductors and bad conductors. Generally speaking, all metals are good conductors, although some are better than others; gases, on the other hand, are very bad conductors of heat.

For purposes of comparison we use a quantity which is called the "thermal conductivity" of a substance. It is the number of units of heat which, in unit time, will pass across a cube of unit edge, when the opposite faces are at temperatures differing by one degree.

Thus, in the C.G.S. system it is the number of calories of heat which will pass in one second across a cube of one centi-

metre edge when the opposite faces are at temperatures differing by 1° C.

The following table gives the thermal conductivity in C.G.S. units for a number of substances.

Substance.	Thermal Conductivity.
Aluminium	0·48
Copper	0·72
Gold	0·75
Iron	0·2
Lead	0·08
Magnesium	0·38
Mercury	0·02
Nickel	0·14
Platinum	0·19
Silver	0·96
Tin	0·15
Zinc	0·26
Cement	0·0007
Cotton Wool	0·00004
Ebonite	0·00037
Flannel	0·000035
Glass	0·0018
Water	0·0014
Air	0·000057
Hydrogen	0·00032
Oxygen	0·000056

It will be observed that, compared with the metals, even the worst of them, water is a very bad conductor of heat.

Experiment 60.

Load a small piece of ice with a strip of sheet lead so that it will sink into water. Drop it into a test-tube nearly full of water.

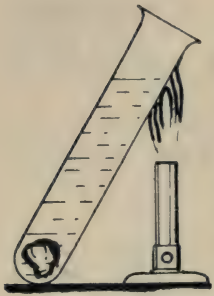

FIG. 94.

Having supported the test-tube in a slanting position, apply a small flame of a Bunsen burner near the top of the tube, just below the surface of the water, as shown in Fig. 94.

Heat can only reach the ice by conduction through the water or the glass, and since the latter is very thin the majority of the conduction takes place through the water. It will be found possible to boil the water at the top of the tube for some considerable time before the ice melts, thus showing water to be a very poor conductor.

Experiment 61.

Convection Currents.—Support a beaker, half full of water, on a tripod with the small flame of a Bunsen burner beneath it. A few fragments of some light material should be put into the water; tiny pieces of red blotting-paper are very suitable.

As the water is heated it will be observed that there is an upward current of water immediately above the flame, while near the outer edge there is a downward current. These movements of the water will be rendered visible by the motion of the particles of blotting-paper which have been put into the water. Fig. 95 shows the general arrangement.

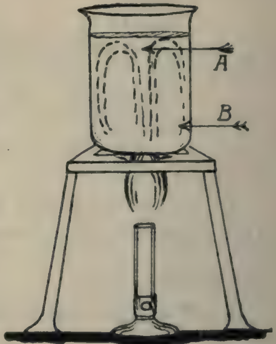

FIG. 95.

If a thermometer is supported with its bulb at the point marked *A*, while another thermometer has its bulb at *B*, a considerable difference of temperature will be noticed.

It will be seen that the heat of the flame passes through the glass by conduction and heats the film of water in contact with the glass. Expansion necessarily takes place which renders the warm water lighter than the surrounding water.

Hence the warm water rises in a constant stream up the middle. The colder water at the edges has no option but to flow in and take its place.

This in turn gets heated and rises, and so the circulation continues. These currents are known as "convection currents."

It will be observed that in the case of *conduction* of heat, the heat is passed on from particle to particle. With *convection*, however, a particle absorbs a certain amount of heat and then moves off, carrying the heat with it.

Domestic Hot Water Systems.—In many kitchen ranges there is fitted a small iron boiler (*B*, Fig. 96), between the grate and the flue. One pipe passes from the top of this to the storage tank (*C*) in the upper part of the house. The tank is generally connected with a constant level cistern.

As the water in the small boiler becomes hot it flows up one pipe (shown black) to the storage tank *C*, while cold water flows from the latter, down the other pipe, into the boiler.

Here we have circulation maintained by means of convection currents. Taps are fitted, where needed, on the "hot" pipe, that is, the one up which the hot water flows.

A general arrangement of such a system is shown in Fig. 96. Of course, convection

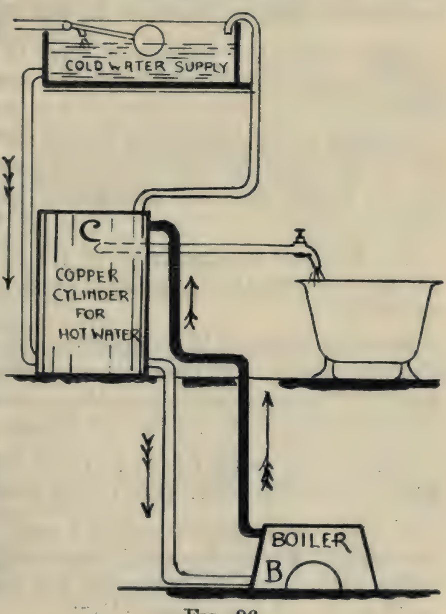

FIG. 96.

currents can be produced in any fluid. Winds are largely convection currents in the atmosphere and many forms of ventilation are an application of this phenomenon.

Formation of Ice on Water.—It will be convenient here to revert to the effects of the abnormal behaviour of water under change of temperature. Consider a river in frosty weather. As the water cools it contracts, and because it is specifically heavier than the water beneath it, it sinks to the bottom and the warmer water rises to the surface, and is cooled in its turn.

After a time the whole of the water is at a temperature of 4° C. Now it will be recalled that at this temperature water attains its maximum density. Hence, as the surface water is cooled still further it *expands*, and this, making it specifically lighter, causes it to remain on the surface.

After a time the uppermost layer of water has been cooled to 0° C., and on the withdrawal of more heat a film of ice is formed.

As soon as the surface of the water is covered with a film of ice, no more ice can be formed except by the withdrawal of the latent heat by conduction through the ice already formed.

The thermal conductivity of ice is 0·0022, and hence we see why it is that a prolonged frost is necessary to produce thick ice.

If water behaved in a manner similar to that of most other liquids, that is, if it contracted as the temperature fell right down to freezing point and then contracted still further on solidification, ice would be specifically heavier than water.

This would cause ice to form at the bottom of the rivers first and solidification would then take place *upwards*.

If such conditions prevailed a quite ordinary winter would be sufficient to freeze most rivers solid. Experiment 60 will show with what difficulty the ice would be melted by the application of heat from above.

Experiment 62.

Half fill a test-tube with paraffin wax cut up in pieces about the size of a pea.

Very slowly melt the wax by the application of heat.

Note that the solid material is heavier than the liquid, since it sinks in the liquid.

Now allow the wax to cool until it is solid again. Note how the surface shrinks down as the solidification proceeds, showing that a marked contraction occurs on passing from the liquid to the solid state.

Articles made of cast iron are always more or less rough owing to the sand used, but to prevent the molten iron from shrinking away from the mould, all large castings are "fed." The slowly-cooling iron is forced into the mould by an iron rod worked up and down.

The pattern-maker has to make the pattern somewhat larger than the required size of the casting because the liquid iron contracts on solidification.

Radiation.—So far as we have considered the question, whether heat is transferred by conduction or by convection currents, matter in some form is necessary for the process.

Radiation is the name given to a process by which heat can travel without the aid of matter in any form.

Radiant heat, as it is called, is a transverse wave motion exactly similar to that of light. In fact, radiant heat is sometimes spoken of as "invisible light."

Heat rays may be reflected, refracted, transmitted and absorbed exactly like light rays.

If we make a bar of iron white hot it emits light rays like any other source of light. It also emits *heat* rays, and although they do not affect the eye in the same way as light, they can be *felt*.

If the iron were heated to a temperature of 200° to 300° C. it would not emit any light rays, but the hand placed at a distance of a few inches would be able to detect the presence of heat rays.

The heat which we feel from a fire is very largely due to radiation. Air, as we have seen, is a very bad conductor of

heat, and the convection currents which are set in motion by the fire cause the hot gases to flow up the chimney. Hence very little heat reaches us from the fire by either conduction or convection.

Experiment 63.

Obtain a bright tin canister and make a mark on the inside about three-quarters of the distance up.

Stand the tin on a block of wood and pour boiling water into the tin up to the mark.

Place a thermometer, with its bulb in water, and record the temperature every half minute. Plot a graph showing the relation between time and temperature.

A little thought should make it clear that the slope or steepness of the curve is an indication of the rate of cooling.

The tin (and similar tins) should now be treated in different ways. One may be painted white, another black; some may be lagged with paper of different thicknesses.

Cooling curves should be obtained for each case and from these an opinion may be formed as to the best methods of encouraging or preventing radiation, according to the needs of the case.

Experiment has shown that the rate of radiation depends upon :—

(1) The difference between the temperature of the radiating body and that of the surrounding bodies.

(2) The area of the surface from which the radiation is taking place.

(3) The nature of the radiating surface.

Generally speaking, a smooth and shiny surface is a bad radiator, while a rough surface is a good radiator. White surfaces are not such good radiators as black, other things being equal.

Experiment 64.

Melt the wax in the test-tube used in Experiment 62 and place the bulb of a thermometer in the liquid. Having supported the tube so that it is suspended in still air, take the

temperature every half minute until the wax has all solidified. Express the results graphically.

Fig. 97 shows the graph of a typical set of results. Observe that the left-hand portion of the curve is very steep. Here we have the hot liquid radiating its heat very quickly, since the difference of temperature between it and the surrounding objects is considerable.

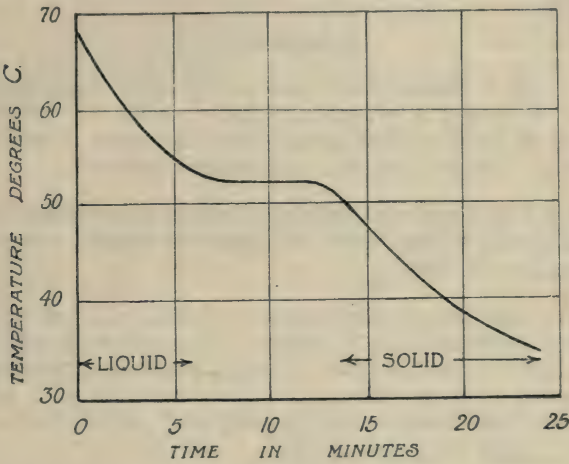

FIG. 97.

The last portion of the curve is less steep. Here we have a solid whose temperature is not very much above that of its surroundings. Hence the cooling is taking place more slowly.

In the middle of the curve we have a flat portion where the temperature remains constant for some minutes.

Now although "cooling" ceased during this period, there is no reason to suppose that radiation ceased also. During the interval the paraffin was turning from the liquid to the solid state, and its latent heat had to be disposed of.

Hence it is reasonable to assume that there was no interruption in the process of radiation. But for the first seven or eight minutes the radiant heat was supplied by the sensible

heat of the liquid paraffin. For the next eight or nine minutes the latent heat of the solidifying liquid was radiated.

It is only after the whole of the liquid has solidified that cooling, as we generally understand it, takes place again. The sensible heat of the solid paraffin is then radiated.

The temperature at which the graph flattens (in this case 52° C.) is the melting point of the wax.

Exercises 17.

1. A rod 1″ diameter is made partly of iron and partly of wood. If paper is wrapped tightly round the rod and a Bunsen flame is passed along the rod, state what will be observed. What does this experiment prove?

2. On a cold morning the fitter finds that his chisel appears to be much colder than his hammer handle. Give the explanation of this.

3. An iron tube, 0·95″ diameter and 15″ long, had a number of brass fins fixed to it, 2″ square. The tube was heated and then allowed to cool and the following readings of temperature in degrees F. were taken with time intervals of 1 minute. Plot time and temperature, heating and cooling curves.

Heating curve, 1 *minute intervals* :—

61, 71, 79, 88, 99, 112, 124, 137, 150, 156, 164, 175, 184.

Cooling curve, 1 *minute intervals* :—

150, 139, 129, 126, 122, 118, 114, 111, 108, 105, 102, 98, 96, 94, 92, 90, 88, 87, 86, 85, 84, 83, 82, 81, 80, 79, 78, 76, 75.

4. The following results give data for the heating and cooling curves of a saturated solution of salt and water. One minute interval was allowed between each reading. Plot time and temperature (Centigrade) for both the heating and cooling curves.

Heating curve :—

14, 18, 28, 37·5, 47, 56, 65, 74, 81, 88, 91·5, 106, 106·5, 107, 107.

Cooling curve :—

104, 102, 101, 100, 98, 96, 94, 92, 90, 88, 86, 84, 82, 80, 77, 74, 72, 69·5, 68, 65·5, 63, 61, 59, 57·55, 53, 52, 51, 50.

5. Steam has to be carried by piping a considerable distance away from a boiler. Explain why it is necessary to lag the pipes.

6. Give sketches showing the methods of cooling the cylinders of motor cycle engines. On your sketches note whether the heat is conducted away by radiation, conduction, or convection.

7. Explain why an eiderdown quilt makes a good bed-covering in winter.

8. Apparatus for making ice cream consists of a thick wood bucket and an inner can of thin metal. Explain why.

9. Explain the principles underlying the vacuum-flask.

10. A room is heated by means of an open fire. Explain with the aid of sketches the various ways in which the heat is transmitted to the room.

11. In order to economise fuel, food is sometimes cooked by means of a " hay box." Explain the underlying principles.

12. A series of experiments was carried out by Brill to determine the value of various commercial coverings of steam pipes. A length of piping 60′ long was used and the heat loss was determined by the condensed steam. The following are some of the results :—

Kind of covering.	Thickness of covering.	B.Th.U. radiated. per sq. ft. per min.
Bare pipe		12·27
Magnesia	1·25″	1·74
Mineral wool	1·30″	1·29
Fire felt	1·30″	2·28
Hair felt	0·82″	1·91

Explain how heat is lost from steam pipes and state what you can deduce from the above figures.

13. A cooling curve was taken for melted tallow. Temperatures were measured in degrees Centigrade, and time intervals of one minute were allowed between each reading. Plot a graph of time and temperature.

Cooling of melted liquid.—145, 140, 140, 134, 128·5, 122, 116, 110, 105, 100, 95, 90, 85·5, 81, 76, 72, 68, 64, 60, 58, 56, 52, 50, 47·5, 45, 42·5, 40, 38·5, 37; solidification starts; 35, 33·5, 33, 32, 31, 30·5; thin layer of solid crust; 30, 30, 29·5, 29, 28, 27·5, 27, 27, 26·5.

14. From the following figures, plot a graph, showing how the weight of water varies with temperature :—

Temperature.	Weight in lbs. per cub. ft.
32° F.	62·42
52	62·4
62	62·36
72	62·3
82	62·21
92	62·11
102	62·0
122	61·7
142	61·34
162	60·94
182	60·5
202	60·02
212	59·76

15. Give sketches of a small chamber, suitable for providing cold storage for a butcher. Explain the principles upon which it works.

16. Tests were carried out with vessels of different metals to see how much steam per hour per square foot they were capable of condensing. The following are some results :—

Iron, 7·5. Brass, 12·5. Copper, 14·5.

What can be deduced from these figures?

17. In a test to determine the specific heat of mild steel it was found that the water equivalent of the calorimeter was 14·2 grammes, 121·685 grammes of water at a temperature

of 17° C. were placed in the calorimeter and 38·85 grammes of mild steel at a temperature of 100° C. were dropped into the water and the water rose in temperature to 20° C. What is the specific heat of the mild steel ?

18. Define the " thermal conductivity " of a substance.

19. It is known that the freezing point of a salt solution is lower than that of pure water. Will this fact help you to explain why salt is often thrown on slippery frozen pavements ?

20. Give reasons to show why lakes freeze more readily than oceans.

21. Kent gives the following figures :—

Per cent. of salt by weight in water—	1	5	10	15	20	25
Freezing point in degrees Fahrenheit—	31·8	25·4	18·6	12·2	6·86	1

Plot a graph of per cent. of salt by weight and freezing point.

22. What weight per cent. of salt will be found in sea water if it freezes at 27° F., assuming sea water to be a common salt solution ?

23. When strong sulphuric acid is poured into a glass vessel containing water it often happens that the vessel breaks, and in any case a big rise in temperature is noted. Can you account for the glass cracking ?

CHAPTER XVIII

THE CONSERVATION OF ENERGY

Energy.—We have on one or two occasions referred to heat as a "form of energy." It is proposed here to consider this idea in a little more detail.

It is a matter of common experience that all the bodies with which we deal offer a resistance to motion. This resistance may be due to the gravitational force of the earth (if any vertical motion is attempted), or it may be due to friction between the body and its support.

In any case the resistance is there, and we define "force" as that which *tends* to overcome resistance.

It should be observed that a force may be applied without overcoming the resistance. For example, a force of 10 lbs. applied to a railway truck would be insufficient to move it. That is, the resistance in this case would not be overcome.

When resistance *is* overcome, we say that "work" is done, and the *amount* of work done is indicated by the product of the magnitude of the force and the distance through which it acts.

Now "energy" is defined as that which is capable of doing work. There are a great many forms of energy. A moving weight is capable of doing work, and is said to possess "kinetic energy." A weight raised to a height above the earth's surface (*e.g.* a lake on a mountain side) possesses what is called "potential energy."

Heat and electricity are forms of energy. The former can be made to do work by means of a heat engine, while the latter can operate an electric motor and thus do work.

A piece of coal possesses what is called "chemical energy." When it burns it combines chemically with the oxygen of the air and heat is produced.

Now any form of energy is (under suitable conditions) transformable into any other form. But there is in every case what we may call a fixed rate of exchange.

The amount of energy in a body is measured by the amount of work which it is capable of doing, and this is in all cases a definite and calculable quantity.

The Mechanical Equivalent of Heat.—It is a common saying that friction produces heat. It would be more correct to say that the overcoming of friction produces heat. Here

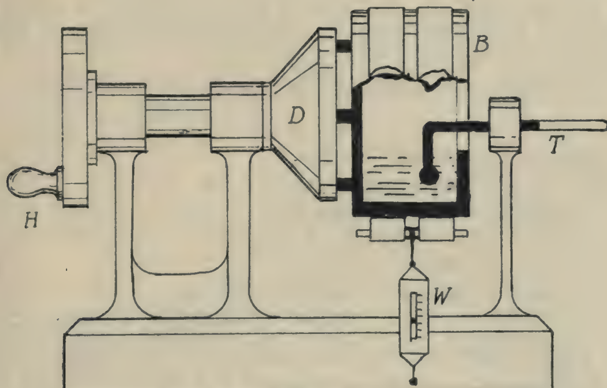

FIG. 98.

we have an example of kinetic energy being converted into heat.

The following is a description of an experiment in which the amount of friction overcome (*i.e.* the work done) is measured and also the number of units of heat produced.

There are several forms of apparatus available. Fig. 98 shows one of a very simple design. It consists of a hollow metal drum D which can be rotated by the handle H.

Passing round the outside of the drum is a band of silk B. One end of this band is held down to the base of the instru-

ment by means of a small spring balance and the other end supports a hanging weight-carrier.

The drum is rotated so that it passes under the silk band in a direction passing from the weight carrier to the spring balance. A little thought will make it clear that the resistance offered at the surface of the drum is equal to the difference between the weight supported and the reading of the spring balance.

Engineering students will recognize this arrangement as a miniature rope-brake dynamometer, as used for the determination of the horse-power of small engines.

A known quantity of water is placed inside the drum and a specially bent thermometer is supported so that its bulb is always in the water.

An example will illustrate the mode of operation.

Ex. 27.

Weight supported, 7·3 lbs.
Reading of spring balance, 0·7 lbs.
Diameter of drum, 6 inches.
Number of revolutions, 107.
Weight of drum, 1·27 lbs.
Weight of drum and water, 1·955 lbs.
Temperature rise of water, 1·8 degrees Fahrenheit.

It will be observed that the friction of the "brake" on the rim of the drum has produced heat and has raised the water (and the drum, too, of course) through 1·8 degrees Fahrenheit.

We will consider the heat first and will estimate it in British Thermal Units.

Heat gained by water :—

$=$weight of water $\times$ degrees rise.
$=(1{\cdot}955-1{\cdot}27)$ lbs. $\times\, 1{\cdot}8$ degrees.
$=\cdot 685$ lbs. $\times\, 1{\cdot}8$ degrees.
$=1{\cdot}23$ B.Th.U.

Heat gained by drum :—

$=$Weight of drum $\times$ degrees rise $\times$ specific heat.

(Now the drum was made of brass, which has a specific heat of 0·094),

Hence heat gained by drum :—

$=1{\cdot}27$ lbs. $\times 1{\cdot}8$ degrees $\times 0{\cdot}094$.

$=0{\cdot}215$ B.Th.U.

Total heat produced $=1{\cdot}23+0{\cdot}215$.

$=1{\cdot}445$ B.Th.U.

Force exerted as friction by the brake $=7{\cdot}3-0{\cdot}7$ lbs. $=$ $6{\cdot}6$ lbs.

And this force acted through a distance equal to the circumference of the drum multiplied by the number of revolutions made.

That is, the distance was :—

$\pi \times 6 \times 107$ inches.

$=168$ feet.

But work done is force $\times$ distance :—

$=6{\cdot}6$ lbs. $\times 168$ feet.

$=1108$ ft.-lbs.

Hence we see that :—

$1{\cdot}445$ British Thermal Units of heat are equivalent to 1108 ft.-lbs. of molar energy.

$$\therefore 1 \text{ B.Th.U.} = \frac{1108}{1{\cdot}445} = 767 \text{ ft.-lbs.}$$

This quantity is called the mechanical equivalent of heat. Of course the instrument described above does not give the most reliable results, as it lacks the somewhat complicated refinements of the more accurate instrument. The principle involved is, however, the same in all cases.

The best results give a value to the mechanical equivalent of heat of 778 ft.-lbs. per B.Th.U. or $426{\cdot}9$ kilogramme-metres per calorie.

A kilogramme-metre is, of course, the work done by a force equal to the weight of 1 kilogramme acting through a distance of 1 metre.

The Conservation of Energy.—We have seen that 1 British Thermal Unit of heat on conversion into kinetic energy, produces 778 foot-lbs. These two amounts of different

types of energy are interchangeable. It makes no difference whether heat is being converted into kinetic energy or *vice versa*, the equivalent is never varied. Nor is it affected by the mode of transformation.

Of course our ignorance or carelessness may result in some of the energy reaching a destination which is not intended.

For instance, in the experiment described in Example 27, it is quite possible that some of the heat produced by the brake was radiated into the air, and therefore failed to reach the water, where it would have been measured.

Again, a steam engine converts heat into kinetic energy, but some of the heat produced in the furnace passes up the chimney-stack, and some remains in the exhaust steam, not to mention other means of escape. Hence only a portion (and in this case a very small portion) of the heat produced is actually converted into kinetic energy.

In a modern steam engine about 10 per cent. of the heat produced in the furnace is converted into useful energy. The remaining 90 per cent. is *there*, but at present we do not know how to use it.

The 10 per cent. in this case is called the "thermal efficiency" of the engine. Some types of heat engine (notably the internal combustion engine) have improved on this to some extent. The student should, however, grasp the fact that no amount of mechanical improvement or novel design can make an engine with a thermal efficiency of *over* 100 per cent, and a great advance is necessary before this degree of efficiency is remotely approached.

No mere mechanism can withdraw from a body more energy than it contains.

Calorific Value of Fuels.—If we take 1 lb. of coal and burn it, heat is produced. This heat may be measured with a suitable calorimeter, and the result is known as the calorific value of the coal.

Of course the numerical value depends upon the composition of the coal, but for a given coal it is an absolutely fixed quantity. The number of thermal units produced

does not depend upon the rapidity of burning, so long as the coal is completely burned.

The following table gives the calorific values of a few common fuels :—

Fuel.	Calorific Value in B.Th.U's. per lb. of fuel.
Coal, Average composition .	14,000
Coke ,, ,, .	13,000
Petrol ,, ,, . .	19,800
Petroleum (commonly called paraffin)	20,100
Hydrogen	62,000
Carbon	14,500

We sometimes see advertisements of patented preparations which claim that when these are mixed with coal the latter will give twice as much heat as it otherwise would. No preparation can possibly produce this result, as it is contrary to the laws of nature.

Such treatment may cause the coal to cake together in the grate and thus last longer, that is, to burn more slowly, but this is quite another thing.

If 1 lb. of coal is burnt in 10 minutes it is producing heat at the rate of 84,000 B.Th.U.'s per hour.

If it takes half an hour to burn the heat is produced at the rate of 28,000 B.Th.U.'s per hour.

In both cases, however, the 1 lb. of coal produces its 14,000 B.Th.U.'s. No more and no less.

Electricity is a form of energy. If it is passed through a thin wire which offers a high resistance the wire becomes sensibly hot. Here we have electrical energy being converted into heat energy.

It is found that 1048 watts of electricity flowing through a resistance for 1 second produce 1 British Thermal Unit of heat.

Again, if electricity is passed through an electric motor, the latter revolves and may be made to do work. Here we have electrical energy being converted into kinetic energy.

It is found that 1 watt of electricity flowing for 1 second, if wholly converted into kinetic energy, is capable of doing 0·742 ft.-lbs. of work.

Hence we have :—

1 watt-second of electricity is equivalent to 0·742 ft.-lbs. of work.

∴ 1048 watt-seconds of electricity are equivalent to $1048 \times 0{\cdot}742$ ft.-lbs. of work. $=778$ ft.-lbs.

But 1048 watt-seconds of electricity are equivalent to 1 B.Th.U. of heat.

∴ 1 B.Th.U. of heat is equivalent to 778 ft.-lbs. of work.

This will be recognised as the mechanical equivalent of heat already mentioned.

We see, therefore, that energy may be transformed from one form to another. This may be done in a great variety of ways, but there is no known method whereby energy may be created out of nothing. Nor can we destroy energy, that is, turn it into nothing.

Efficiency.—Of course, we are seldom able to effect a complete transformation of the kind which we desire.

Thus, if we deliver 1048 watt-seconds of electricity to any motor we shall get rather less than 778 ft.-lbs. of work from the motor, because some of the electricity will be transformed into heat, and in that case will not be available for transformation into kinetic energy also.

Suppose in the case considered the motor is found to give 645 ft.-lbs. of work. We see that electricity equivalent to 133 ft.-lbs. has been lost (*not* destroyed). Perhaps it has become heat.

In any case the ratio of work obtained to work supplied is $\frac{645}{778}=0{\cdot}83$. This is generally expressed as 83 per cent., and is called the "efficiency" of the motor.

It indicates that, of the energy supplied to the motor, 83 per cent. is converted into the form in which we require it.

Exercises 18.

1. Determine the work done in the following cases :—

Load in pounds . .	121·5	89	356·9	302·6
Vertical distance moved in feet	29	32	52	9·2

2. A tug-of-war is in progress and for three minutes each side pulls with an exactly equal force of 495 pounds. How much work is done ?

3. A stone weighs 225 pounds and a man pulls at the stone for 2·4 minutes with a force of 110 pounds and fails to move it. How much work has the man done ?

4. A man removes four shovels full of sand from the ground on to a platform 3′ 6″ high. By the aid of sketches and approximations show how much work the man does.

5. A man is sandpapering a board which is 3′ 6″ long and 12″ wide. He rubs in one direction with a force of 23 pounds. If the rubber is 3″ wide, how much work in foot pounds will the man do in order to rub each part of the board once ?

6. A man is filing a piece of metal and on each forward stroke he applies a horizontal force of 15 pounds with a stroke of 6·5″. If he makes 22 strokes, how much work in foot pounds will the man do ?

7. A man stretches up his arms to hold a heavy weight. At the end of 5 minutes he feels tired. If the weight was 37 pounds, state the work done in foot-pounds. Write a short essay showing the difference between the idea of " work " in a popular and a scientific sense.

8. Illustrate with the aid of sketches a number of cases where work is being done by manual labour. In your sketch show the distance moved and the mass. Give approximate answers for the work done in 5 minutes in each case.

9. What do you understand by the term " work ? " State whether work is done in the following cases and give approximate answers :—

(1) boring a hole with a brace and bit.
(2) carrying a load of 7 pounds 7 feet horizontally.
(3) lifting 7 pounds vertically through 3 feet.
(4) A string holding up a weight of 14 pounds.

10. In a test of lifting tackle, the following results were obtained :—

Load lifted in pounds . .	25	50	100	150
Effort to move load . .	$5\frac{1}{4}$	$7\frac{1}{2}$	$13\frac{1}{4}$	19

If the effort moves 20 times as fast as the load, determine the work done by the load and the effort when the load has moved 1·5 feet. Plot a graph of load and effort.

11. In a test on pulley blocks it was found that the effort moved 4 times as fast as the load. Assuming that the load moved 4 feet, determine the foot-pounds of work done in moving the load and the effort through their respective distances. Plot curves of load and effort. The following are the figures required :—

Load in pounds—	5	10	15	20	25	30
Effort in pounds—	3·8	6·3	8·9	11·0	12·6	14·7

12. In a worm gearing it was found that the effort moved 31 times as fast as the load. If the effort moved one foot, determine the number of foot-pounds of work utilized in moving the load through one foot. Plot a graph of load and effort.

Load in pounds . .	4	6	8	10
Effort in pounds .	0·3	0·5	0·75	0·9

13. The following calorific values in B.Th.U. per lb. are given :—Texas oil, 10,800 ; Trinidad crude, 10,200 ; shale oil, 10,120 ; heavy tar oil, 8,916. Convert these values into calories per gramme and equivalent foot-pounds of work.

14. The following theoretical maximum combustion temperatures are given :—Natural gas and air, 1,086° C. ; Thermit, 2,694 C. ; oxyhydrogen flame, 3,190°C. Convert these values into Fahrenheit degrees.

15. In a determination of the electric horse-power of a motor the following observations were made :—

Volts	490	490	490	490
Amps. . . .	1·8	2·2	2·6	2·8

Since amps. × volts = watts, determine the number of foot-pounds of work done per minute in each case.

16. In a further test on the above motor, the brake horse-power was found to be as follows :—0·48, 0·73, and 0·95. If 33,000 ft. lbs. per minute is equal to one horse-power, determine the foot-pounds of work per minute in each case.

17. In a gas engine test the following heat distribution was obtained :—

Work obtained as indicated horse-power .	21·5%
Lost in the jacket water	50·4
Lost in the exhaust gas	25
Balance of the losses	3·1
	100

If the total indicated horse-power is 2·3, determine these ratios as foot-pounds of work per minute.

18. The average heating value of the following gases is given in B. Th.U. per cubic foot :—Illuminating gas, 600 ; coke oven gas, 650 ; producer gas from coke, 135 ; blast furnace gas, 100. Determine the equivalent values in foot-pounds of work.

19. An engine is directly coupled to a generator and it is found that the engine develops 50 indicated horse-power. Neglecting all losses in engine and dynamo, determine the foot-pounds of work and the watts generated per second. If the work available from the engine is only 80 % of the I.H.P. and the efficiency of the generator is 95 %, how many watts are generated per second ?

20. Wherever energy is used to produce motion, some of the energy is wasted. Write a short essay, illustrated by sketches, showing cases where energy is being wasted, and explain what becomes of the wasted work.

21. An experiment was carried out to determine the mechanical equivalent of heat and the following are the results :—

Weight of bottom cup of copper, 41·15 grammes.
Weight of top cup and water, 71·25 grammes.
Weight of top cup, 55·4 grammes.
Original temperature of the water, 17·5 C.
Final temperature of water 30·2 °C.

To cause this rise of temperature the apparatus made 2,420 revolutions, a load of 0·155 lbs. being applied to the rim of a wheel whose circumference is 800 millimetres. Determine the mechanical equivalent of heat in ft.-lbs. per B.Th.U.

22. In another experiment performed with a similar piece of apparatus the following results were obtained :—

Weight of two brass cones, 164·1 grammes.
Weight of contained water, 21·35.

Suspended mass, 300 grammes at the rim of a wheel whose circumference was 78·4 centimetres. The total number of revolutions was 1,852 and the temperature rise was 24°C. Determine the mechanical equivalent of heat.

23. Zinc is dissolved by the action of dilute sulphuric acid, and the final product is zinc sulphate, hydrogen, and water. The chemical energy of the products is less than that of the original substances. In carrying out this experiment explain what you have noticed. Where has the lost energy gone ?

ANSWERS

Exercises 1.

1. 26·19, 52·39, 77·79 mm.
2. $\frac{1}{8}''$, $\frac{3}{16}''$, $12\frac{1}{8}''$.
3. 3·609, 4·921, 12·795 ft.
4. 0·186, 0·403, 1·506 sq. in.
5. 0·4989, 1·1340, 2·1319 kgms.
6. 11·24, 14·33, 16·98 lbs.
7. 48, 89·6, 118·4 pdls.
8. 88 ft. per sec., 2680 cms. per sec.
9. 1026 ft. per min.
12. 2640 ft. per min. ; 80400 cms. per min.
13. 45 ft. per sec.
14. 60 miles per hr. per hr.
    0·024 ft. per sec. per sec.
    0·74 cms. per sec. per sec.
15. $3{\cdot}09 \times 10^7$ dynes per sq. cm.
16. $6{\cdot}9 \times 10^5$ dynes per sq. cm.
17. 32 ft.-lbs., 1025 ft.-pdls., $4{\cdot}3 \times 10^8$ ergs.
18. 55·43, 33·43.

Exercises 2.

1. 6·859, 13·824, 79·507, 0·9112 c.c.
2. 0·007854, 1·131, 8·55, 15·90 cub. in.
3. 0·000523, 0·0335, 8·181, 344·8 cub. in.
4. 1·45.
5. 0·283 lbs. per cub. in., 49·17 c.c., 385·5 gms., 7·85.
6. 0·283, 0·284, 0·283 lbs. per cub. in.
7. 0·284, 0·283, 0·283 lbs. per cub. in.
8. 0·361, 0·361, 0·361 lbs. per cub. in.
9. 1, 2·59, 6·68, 9·82 gms. per c.c.

10. 62·5 lbs. per cub. ft., 1 gm. per c.c.
11. 2.57, 8·5, 8·9, 7·7.
12. 2·5, 2·6, 19·26, 7·69, 10·5.
13. 0·283, 0·277, 0·314, 0·297 lbs. per cub. in.
14. 0·67, 0·75, 0·72, 0·74, 0·66.
15. 96, 115·3, 49·5, 55·6 lbs.

Exercises 3.

1. 1·14 gms. per c.c.
2. 8·55 ,, ,,
3. 0·859 ,, ,,
4. 13·59 ,, ,,
5. 13·47 ,, ,,
6. 13·61 ,, ,,
7. 7·82 ,, ,,
8. 0·52 ,, ,,
9. 0·818, 1·22, 0·87.
10. 1·06 gms. per c.c.
11. 0·8.
12. 1·34.
13. 0·78.
15. 2·41 gms. per c.c.

Exercises 4.

1. 1·11 lbs. per sq. in.
2. 0·039 lbs. per sq. in.
3. 4·3 lbs. per sq. in.
4. 14·5, 2·2 lbs. per sq. in.
5. 26·5″.
6. 0·0141, 0·0154, 0·0159 lbs. per sq. in.
7. 1798.
9. 0·014 lbs. per sq. in.
15. 0·144″.
16. Less than 46 lbs.
19. 0·491.
20. 29·9, 26·55, 23·45, 20·6, 18·15 inches.
21. 559, 564, 569, 574 mm.
91·64°, 91·87°, 92·11°, 92·34° C.
22. 2·95, 3·55, 4·14, 4·73, 5·32 inches.

23. 2·73 cub. in.
24. 83·4 cub. in.
25. 14·22, 14·35, 14·67, 14·80, 14·90, 15·08 lbs. per sq. in.
26. 0·99909, 0·99989, 1·00005 gms. per c.c.
27. 50 cub. ft.
28. 26·1 ft.

Exercises 5.

4. 0·65 sec.
5. 1·25 sec.
6. 0·55 sec.
14. ·2 secs.
15. 9·8″.
16. 0·073″

Exercises 6.

9. 329, 1493, 251, 507, 5340, 3165, 3352 metres per sec.
12. C′.
13. Middle C. 14·9 revs. per sec.
14. $\frac{283}{256}$, $\frac{5}{4}$, $\frac{341\cdot3}{256}$, $\frac{3}{2}$, $\frac{213\cdot3}{128}$, $\frac{15}{8}$, 2.
15. E, G. C′.
16. $C_{\prime\prime\prime}$, $A_{\prime\prime}$, E.
18. Middle C.
20. 200 feet per sec.

Exercises 8.

1. $\frac{4}{3}$, 2, 4.
4. 41·9 ft.
5. 1620 yds.
6. 0·416, 14·7 secs.
16. 336·5, 346, 349 metres per sec.

Exercises 9.

3. 0·0000004861, 0·0000004308, 0·0000005167, 0·000000382 metre.
4. 0·0000486, 0·00008, 0·0002, 0·00025 cm.
6. 8·3 minutes.
8. Circle of 3″ diameter.
Vertical line 3″ long.
11. $\frac{3}{16}$″ or $\frac{1}{2}$″.
15. 32·7, 26·7, 0·9, 20·4, 27·6.
17. 0·093.

Exercises 10.

7. 60°.
8. 0·083, 0·125, 0·127, 0·318.
14. 40°.

Exercises 11.

9. 140300, 137200, 124400, 111700 miles per second.
15. $1\frac{1}{2}$ ft.
16. 2·46, 3·8, 4, 3·57, 4·5 inches.

Exercises 12.

5. 150, 150, 160, 200, 360 mm.
6. 150, 150, 160, 200 mm.
7. 7·3, 7·1, 4·88, 20·9, 21·2, 15·7, 14·2 cms.
10. 19·275 cms. 22·742 cms.
16. 15 cms.
18. 25·6 cms.

Exercises 13.

13. 1·7° C.
14. 36·9° C., 20° C.
15. 1094, 1148, 1310, 1472, 1687, 1742, 1994, 2040, 2282° F.
863, 893, 983, 1073, 1193, 1223, 1363, 1423, 1523° A.
16. 1051, 1175, 1375, 1549, 1650, 1726, 1771, 2201° F.
17. 156, 430·5, 218, 61.
18. Pressures: 0·95, 1, 2·04, 6·1, 6·8 atmospheres.
Temperatures: 98·6, 100, 121·4, 160·3, 164·4°, C.
19. −188° F., −182° F., −229°F.
706, 821, 494 lbs. per sq. in.
21. 224° C., 299 C., 316° C., 427 C.

Exercises 14.

1. 8·0005″, 8·001″.
2. 4·9998″, 5·0003.
3. 2·0004, 4·0009 6·0013, 8·0017.

4. 6·012″.
5. 9·9986 ft.
6. 0·26″
7. 0·0018 inch per foot
8. 0, 0·0002, 0·00045, 0·0007.
9. 0·00084″.
10. 0·000347″.
11. 0·013 per cent.
15. 0·136″.
21. 0·148″.
24. 1·45″.
26. 0·037 cub. in.
29. $\frac{1}{30}$.
32. 0·0000186.
33. 0·000167.
34. 0·004.
35. 0·00365.
41. 165 c.c.
42. 29·5 c.c.
43. 140.

Exercises 15.

1. $2{\cdot}72 \times 10^5$, $1{\cdot}37 \times 10^6$, $1{\cdot}45 \times 10^6$, $1{\cdot}93 \times 10^6$ calories.
272, 1370, 1450, 1930 grand calories.
2. 1080, 5440, 5740, 7650 B.Th.U.
5. 4230, 2200, 2490 B.Th.U.
6. 5660 B.Th.U. per hour.
7. 3·94, 3·33, 3·05, 2·83 B.Th.U.
993, 840, 768, 713 calories.
8. $5{\cdot}11 \times 10^6$, $5{\cdot}01 \times 10^6$, $5{\cdot}03 \times 10^6$, $4{\cdot}69 \times 10^6$ calories per lb.
9. $1{\cdot}24 \times 10^6$, $4{\cdot}18 \times 10^6$, $2{\cdot}1 \times 10^6$, $3{\cdot}21 \times 10^6$ B.Th.U. per lb.
10. 0·168, 0·0818, 0·0226, 0·186, 0·045, 0·069.
12. 0·34 lb. 3 B.Th.U.
13. 350·97 calories.
14. 0·14.
15. 11·4, 11·3.
16 0·1.
18. 39·5° C.
19. 38,100 B.Th.U.
20. 282, 515, 244 B.Th.U.
21. 183·5 lb.-deg. Cent. units.
22. 11200, 14540, 13600 B.Th.U.
24. 35·3 lbs. per sq. in.

Exercises 16.

6. 977 B.Th.U. per lb.
7. 79·1.

8. 518.
16. 1405, 875, 106 B.Th.U.
0·019, 0·039, 0·014.
17. $6{\cdot}74 \times 10^6$ B.Th.U., $1{\cdot}6 \times 10^7$ B.Th.U.
0·42.
22. 0·604, 0·864.
23. 9176 B.Th.U.
24. 63° C.

Exercises 17.

17. 0·13.
22. 4 per cent.

Exercises 18.

1. 3525, 2848, 18550, 2780.
5. 322 ft.-lbs.
6. 178·7 ft.-lbs.
10. Work on load :—37½, 75, 150, 225 ft.-lbs.
Work by effort : 167½, 225, 397½, 570 ft.-lbs.
11. Work on load :—20, 40, 60, 80, 100, 120 ft.-lbs.
Work by effort :—61, 101, 142, 176, 202, 235 ft.-lbs.
12. 0·129, 0·193, 0·258, 0·328 ft.-lbs.
13. $2{\cdot}72 \times 10^6$, $2{\cdot}57 \times 10^6$, $2{\cdot}6 \times 10^6$, $2{\cdot}25 \times 10^6$ calories.
$8{\cdot}4 \times 10^6$, $7{\cdot}94 \times 10^6$, $7{\cdot}88 \times 10^6$, $6{\cdot}94 \times 10^6$ B.Th.U.
14. 1987° F., 4881° F., 5774° F.
15. 39300, 48000, 56800, 61100 ft.-lbs.
16. 15840, 24090, 31350 ft.-lbs.
17. 16330, 38300, 18980, 2290 ft.-lbs. per minute.
18. 466800, 505700, 105030, 77800 ft.-lbs.
19. 27500 ft.-lbs. per sec.
37100 watts, 28150 watts.
21. 784 ft.-lbs. per B.Th.U.
22. 0·36 kgm.-metres per calorie.

INDEX

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